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Teaching Model Construction for Solving Physics Problems

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Recent educational research has shown that, after receiving conventional physics instruction, most students do not develop anticipated scientific reasoning abilities. Three major causes have been reproached:

1. Physics students are often encumbered with naive preconceptions about the world that are incompatible with scientific theory and practice. These preconceptions are not treated adequately in instruction. As a consequence, and because of the conflicting interference of these preconceptions with the learning process, students cannot assimilate scientific knowledge adequately (Halloun 1986, Halloun and Hestenes 1985, McDermott 1984, Clement 1982, Champagne et al 1980).

2. Physics is presented with a mathematical formalism that is often beyond student's competence. Most secondary and college students are unable to realize the conceptual relationships that are represented in equations, graphs and other mathematical representations commonly used in physics. As a consequence, students memorize physics formulas in a rote manner, and reproduce them spontaneously in solving exam problems (Halloun 1988, Gamble 1986, Clement 1981).

3. The paradigmatic* conceptual structure of physics (Kuhn 1970), and physics problem solving strategies are seldom made explicit in conventional instruction. As a consequence, students often do not develop anticipated scientific reasoning abilities, and they solve exam problems by trial error, without understanding necessarily the conceptual structure of these problems (Halloun and Hestenes 1987, Reif 1986, Strand 1986, Arons 1981).

The research reported in this article deals with the remedial of the last deficiency in conventional physics instruction. Strategies that students and physics use to solve physics problems are first contrasted in the article. Then, a dialectic teaching method is proposed that prescribes how to construct appropriate physico-mathematical models for solving physics problems. Two samples of secondary and college students were trained

* I define a *paradigm* as a conceptual system that consists of a theory and all the rules that establish the correspondence of the theory to the real world or govern its deployment in the resolution of the world problems.

following this method. The results of the training are reported and discussed at the end of the article.

I. Naive and Scientific Procedures for Solving Physics Problems

Knowledge of any type is organized in our memory into mental systems whose basic component is the *concept*. A scientific concept is defined by a set of laws and rules (Halloun, in press).

These are:

1. *Rules of correspondence* that establish the utility of the concept, by defining the set of events or beings (real or imaginary, factual or action-type), or a corresponding property that the concept can represent.
2. *Laws of organization* that establish how the concept can be related to other concepts in a given theory.
3. *Laws of quantification* that establish how the concept can be quantified and measured.
4. *Rules of deployment* that establish how the concept can be used to study real world situations, and that govern the activities (mental or physical) that need thereby be undertaken.
5. *Rules of expression*, or a scientific vocabulary that includes the concept name, the names of its dimensions and units, as well as the mathematical symbols that can be used to represent it and relate it to other concepts.

The same type of rules and laws are also used in the definition of a given scientific *model* whose elementary building blocks are concepts.

Laws of organization and rules of expression associated with individual concepts and models are often referred to in the literature as *declarative Knowledge*. Rules of correspondence and deployment and laws of quantification associated with individual concepts and models and general mental and motor skills, needed to undertake a given experiment or solve a given problem in the context of a well-chosen paradigm, are often referred to as *procedural knowledge*.

Declarative knowledge, and especially rules of expression may be accumulated passively, and even in a rote manner. However, procedural knowledge cannot be accumulated but actively and through-practice. Thus, the latter type of Knowledge is at a higher cognitive level than the former, and is harder to accumulate.

Declarative knowledge is often well presented in physics textbooks, whereas procedural knowledge is there often implicitly. That is why, and because of the acute contrast regarding procedural knowledge between students and physicists, we focus, in this article, on this type of knowledge in the learning of physics, and more specifically in problem solving.

Students Knowledge State

Students concepts are often vaguely structured, mutually confused, bound to specific situations, and concentrate on apparent irrelevant structure and behavior. (Halloun & Hestenes 1986, Chi et al. 1981). Their procedural knowledge pertaining to problem solving include only some operational (deployment) rules that allow them to transform given data spontaneously into mathematical equations. These equations are chosen by trial and error, following some rules of thumb, in such a way to engage all given data in the mathematical model constructed to solve the problem (Reif and Heller 1982, Larkin et al 1980).

Students' common strategy for solving textbook problems consists of the following steps:

1. Read a given problem and identify the «unknown variables». This lecture is often done backwards, starting with the questions.
2. Read the givens and identify the "known variables".
3. From a repertoire of formulas memorized in a rote manner, select the formulas that relate the "unknown" variables to those whose values are given.
4. Solve the mathematical system of equations established in the previous step.

Unfortunately, many textbook and exam problems can be "solved" following this strategy, which encourages students to stick to their habit.

Given data seldom require critical analysis. These data are given to be used entirely in the solution of the problem; the student does not have to choose between relevant and irrelevant data.

Furthermore, conventional questions of the type "find" or "evaluate" the magnitude of a given concept that are commonly asked are obvious and straightforward. In contrast to open-ended problems that ask questions of the type "under what condition" can a certain event occur, conventional questions do not require the student to decide first on the concepts that fit best his needs to solve the problem.

Problems with superfluous data or open-ended questions help students develop critical thinking, in addition to rules of correspondence and deployment of various concepts, which is a major objective in teaching science, in general, and physics, in particular. As long as students are not presented with enough problems of this type, they will not be motivated to modify their problem solving strategies.

Physicists Knowledge State

Physicists concepts are well-structured and organized around basic laws in appropriate paradigms. For them, solving problems consists not of

a task of recall of appropriate formulas, but of a task of construction of appropriate physico-mathematical models in the context of a well chosen paradigm. Such task goes in three major steps: a) transformation of raw data into a physical model, b) then, or concurrently, into a mathematical model, and c) exploitation of the model thus constructed.

1. Construction of a physics model

Given a theoretical or an experimental problem, physicists determine first the appropriate paradigm for solving it. Then, they separate the different systems involved in the problem, select relevant data for each system, and represent them by appropriate physics concepts in a well chosen reference system. This kind of analysis is often referred to as the *description* of the problem (Hestenes 1987, Heller and Reif 1984).

If a problem is classified, say, as one of Classical Mechanics of particle-like objects (e. g. in the case of translational motion of macroscopic objects), such objects are first identified, and corresponding inherent and kinematical properties are selected and defined relative to a reference system. Then, the environment of every object is identified, and the corresponding dynamical properties (forces or fields) are defined in the chosen reference system. Physicists often start building the mathematical model at this level and represent the kinematical and dynamical concepts with appropriate vectors in a coordinate system representing the reference system.

Construction of the physics model is made with the explicit use of concepts or models components. Let us stay with the case of objects that can be represented by the particle model: a) Rules of correspondence specify whether an object can be represented by such a model. They also specify what kinematical concepts are appropriate for representing the object's motion, and what dynamical concepts, for representing the interaction with its environment. That environment consists of agents, i. e. bodies that interact with the object in question. b) Laws of organization are given by Newton's laws and the Work-Energy principle. c) Laws of quantification specify whether a concept is scalar or vectorial, constant or variable, and set how to measure its magnitude in an appropriate unit system and how to specify its direction, if it is vectorial. d) Rules of deployment state that the only inherent, relevant property, mass, and kinematical properties must be defined only for particle-like objects and not for their agents (unless the motion of the agents is to be studied), and that only forces exerted by the agents on the objects must be considered. Rules of deployment also help make the strategic choice between Newton's laws and the Work-Energy principle for solving a given problem. That choice is

often based on the time-dependence of the variables studied. Some problems may require the use of both laws and principle. e) Rules of expression specify among others the mathematical symbols and labels that can be used to represent the different concepts, in addition to the correct names of these different concepts and their units.

Once the relevant data are selected and represented, physicists choose the appropriate definitions and laws for relating the different concepts selected above, and they identify boundary conditions and constraints, if they exist in the problem (Hestenes, 1987).

At this level, the physics model would be completed. Physicists then transform this model into a mathematical model. This transformation, we recall, may be made in the course of developing the physics model.

2. Construction of a mathematical model

In this stage, physicists express concepts definitions, laws, constraints, etc. in mathematical symbolism (e. g. equation, graphs, diagrams). Then they identify the goals (questions of a textbook problem), and verify the completeness of the mathematical model thus constructed, i. e. they check whether the relevant data have been entirely and adequately represented mathematically, and whether the mathematical model is sufficient to reach the desired goals.

3. Exploitation of the model

Once the mathematical model is validated, experts solve it, check the results and evaluate them in terms of their correspondence to empirical evidence. Then they analyze these results and interpret them as to their consequences on the systems studies.

The construction of the physics model, and especially the description part of it, is the most critical in solving problems. A trade-off exists between this description and the effort needed to choose the appropriate strategy for solving a problem. The better this description, the less effort will be needed to make that choice. In fact, a good portion of the problem solution originates at this level.

Students often skip this step. For a physics problem is to them nothing but a task for constructing a mathematical model. Consequently, and as long as physics instruction does not show explicitly how to construct physico-mathematical models and exploit them, students will keep believing that physics offers them nothing but some "applications to mathematics".

II. Teaching Scientific Procedural Knowledge to Physics Students

A prescription was developed that shows how to construct physico-

mathematical models. Groups of high school and college students were trained in a dialectic approach to use this prescription in solving Newtonian Mechanics problems. The training was validated both internally, i. e. with regard to its immediate effect on solving mechanics problems, and externally, i. e. with regard to its extended effect on participants' general classroom performance.

Procedure

84 Lebanese subjects participated in this research. 59 were high school students (Grade II), and the rest, college students enrolled in an introductory physics course at a Lebanese university. High school students had finished their academic year with an overall weak achievement (their cumulative annual mean score was about 45/100). They were required to follow a special 5-week training session during the summer of 1987, which included 20 hours of physics, ten of which were devoted to the study of statics. Students were divided into two homogeneous groups, one of 25 subjects, the other of 34. The homogeneity of the two groups was verified by the administration of an appropriate pretest, which was also used to reveal major deficiencies in the procedures subjects used to solve statics problems. The two groups were trained by the author of this article. Unfortunately a control group of high school students was not available to compare its performance to the trained groups'.

College students were science majors enrolled, at a Lebanese university, in a calculus-based introductory physics course whose enrollment list included 107 subjects. 28 of these subjects have chosen voluntarily to follow a 5-period training session during the semester of their enrollment, and just after they had finished the mechanics part of this course. Each period consisted of 100 minutes. Training was given by the author, whereas formal instruction in the course was given by a different professor who was well experienced in teaching such an introductory course. Three of the volunteering subjects, and 30 of the remaining 79 subjects (control group), withdrew from their formal course. All withdrawees were thus excluded in our study. The experimental college group was then considered of 25 subjects and the control group of 49 subjects. A mechanics diagnostic pretest (described in Halloun and Hestenes 1985) was administered to all college students and showed that the experimental and the control groups were homogeneous before the training.

Training Method

A socratic dialectic teaching method was followed in training all participating groups (Halloun and Hestenes 1987, Posner and Gertzog

1982). Subjects participated actively so that they would: a) detect anomalies in their initial knowledge state and realize the limitations of their procedures, b) realize the necessity for changing these procedures, c) appreciate every component of the prescribed scientific procedures, and d) realize the validity of deploying such prescription in other fields.

A number of problems were conveniently chosen to implement the teaching method. Problems used in the training of high school students were concerned with the equilibrium of particle-like objects and rigid bodies. Those used in the training of college students were concerned with rectilinear, circular and vibrational translation of particle-like objects, and made use of both Newton's laws and the Work-Energy principle. That principle was not required in the training of high school subjects. In figures 1 and 2 are samples of problems similar to those used respectively in the training of high school and college students. Problems shown in the figures were administered at the end of the training to assess the internal validity of the method (they were not treated during the training).

Experimental subjects were trained to solve problems similar to the ones shown in the figures following a hierarchical prescription that shows explicitly how to develop physico-mathematical models. Subjects participated actively in the gradual development of the prescription, after they were shown, in a first meeting, the inadequacy of their own procedures.

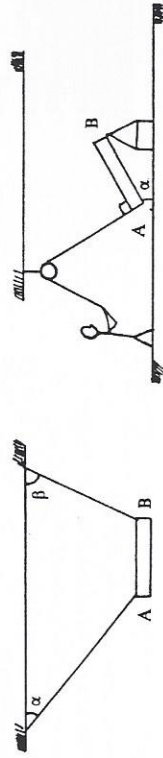


Fig. 1-a

Fig. 1-b

Fig. 1: Study the equilibrium of rod AB

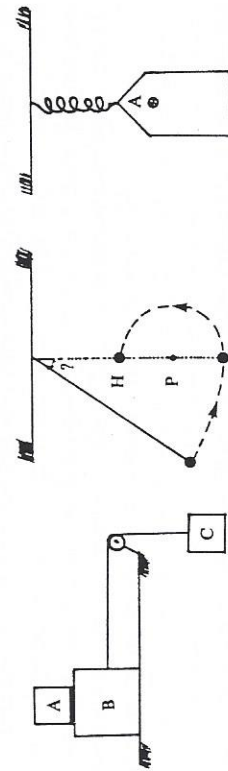


Fig. 2-a: The system is to move with A staying on top of B.

Fig. 2-b: The bob is to reach position H after the string hits peg P.

Fig. 2-c: Apiece of clay is released from A inside the box hanging down the spring.

Fig. 2: Study the motion of the given system.

- The prescription included the following steps;
1. Read the problem and identify its *type* (i. e. the paradigm needed to solve it, which is Classical Mechanics in this case) and *particularities* (translational motion/ equilibrium or rotational motion/ equilibrium).
 2. Identify the real *objects* to be studied, and choose the appropriate physics *model* that can represent each (particle model in the case of translation, rigid body in the case of rotation).
 3. (a) Specify the *reference system* relative to which the problem is to be studied.
(b) Represent that system by an appropriate *coordinate system*.
 4. In that reference system, define:
 - (a) the *inherent* properties of every real object (mass for particles, mass, dimensions and moment of inertia about specific axes for rigid bodies),
 - (b) its *kinematical* properties (position, velocity, acceleration, kinetic energy, ...),
 - (c) the *agents* acting on that object and the corresponding *forces* (or fields).
 5. *Represent* the properties and forces above conveniently in the chosen coordinate system (check, for example, in the case of translation, that the vectorial sum of all the forces in the force diagram is in the same direction as the acceleration).
 6. Choose the appropriate *definitions* and *laws* to relate the different concepts.
 7. Identify *constraints* (initial conditions, limit values, common identical properties for different objects, ...)
 8. Write *equations* (and draw appropriate *graphs*, if necessary) that represent adequately the definitions, laws and constraints identified above. (This step can be undertaken concurrently with the previous two).
 9. Identify *goals*, specify appropriate concepts needed to reach them, and *complete* the mathematical model so that it become sufficient to reach these goals.
 10. *Solve* the mathematical model thus constructed.
 11. Verify that all questions have been answered and check the *validity* of the obtained answers (coherence, clarity, units, numerical values that correspond to empirical evidence...).
 12. *Interpret* the results thus obtained.

Steps 1, 2, 3-a and 4 constitute the description of the physics model which is achieved in steps 6 and 7. Steps 3-b, 5, 8 and 9 make up the mathematical model which is exploited in steps 10, 11 and 12. It goes without saying that this prescription is neither exclusive nor

exhaustive. It is not necessarily to follow it religiously in solving all mechanics problems (especially statics type problems). Through practice, students will be able to judge to what extent they should stick to it in solving various problems.

Results

The proposed teaching method was validated internally for all trained groups and externally for college one. Unfortunately subsequent classroom performance was not available to check the external validity for high school students. The internal validity was assessed using a pretest and a posttest administrated before and after the training. The posttest for every group was made up of the problems shown in the corresponding figures 1 and 2. The pretests consisted of parallel item tests. Neither pretest nor posttest problems were treated during the training.

Table 1: Pretest-Posttest Performance of High School Trainees on Statics Problems*

Task	Percentage of participants**	
	Who did the task correctly on the	
	Pretest	Posttest
Identification of agents & corresponding forces	N / A	80%
Force diagram	20%	69%
Translational Equilibrium equation ($\Sigma F = 0$)	15%	61%
Rotational Equilibrium equation ($\Sigma M = 0$)	10%	53%
Decomposition of forces into components	07%	42%
Resolution of the set of equations	05%	34%

* Those results pertain to problem 1-b in Figure 1.

** Total number of subjects = 59.

Table 1 shows the performance of high school subjects on statics problems before and after the training, following major components of the proposed prescription. A significant improvement can be noticed due primarily to success in the description stage (A t-test done for pre-post training performance, related to the different components shown in table 1, gave values all significant at a p-level that is below 0, 01).

On the posttest, 9 high school trainees (15% of the experimental group) did not identify agents acting on every object explicitly, and 3 others (5%) committed mistakes, in doing so. All these subjects failed to solve the assigned problem correctly (figure 1-b). Of those who were able to identify correctly all agents (the earth, the rope and the wedge), 6 subjects (13%) drew wrong force diagrams, and subsequently made consistent mistakes in writing the corresponding equations of equilibrium. Subjects who were not

able to study the translational equilibrium correctly, were all unable to study the rotational equilibrium correctly and all those who studied the latter type of equilibrium correctly had done so with the translational equilibrium. This shows that mastery of particle mechanics is necessary to understanding rigid bodies.

Table 2: Performance of College Trainees on Particle Mechanics Problems

Task	Percentage of Participants* who did the task correctly in:			
	Rect. Motion**	Circul. Motion**	Vibrat. Motion**	
	Pretest	Posttest	Pretest	Posttest
Motion diagram (Acceleration vector)	N / A	72%	88%	84%
Identification of agents and corresponding forces	N / A	88%	92%	88%
Force diagram	28%	72%	88%	84%
Equation of motion (Newton's laws / Work-Energy principle)	24%	72%***	88%***	76%
Constraints	20%	84%	88%	76%
Resolution of the set of equations	20%	72%***	80%***	76%

* Total Number of participants = 25

** Those results pertain, in Figure 2, to problems 2-a, 2-b and 2-c respectively for rectilinear, circular and vibrational motion. The latter problem required also the use of impulse or conservation of momentum during collision.

*** All those who did the force and motion diagrams correctly.

Table 2 shows college trainees posttest performance on problems of rectilinear, circular and vibrational translation of particles. Pretest performance is also shown for rectilinear motion. A significant improvement can be noticed on this type of problems due primarily to success in the description stage (All corresponding t-values are significant at a p-level that is below 0, 01). Pretraining competence was not assessed for the other two types of translational motion. However, there is no reason to believe that students competence was any better there, especially that these two types of motion are at a higher cognitive level than rectilinear motion.

Table 3 shows that, with respect to classroom performance in their university physics course, the experimental group was significantly better than the control group. Closer analysis showed that this held true for the particle mechanics part of the program as well as for the rest of it (which included rigid body mechanics, waves and fluidmechanics). This shows that

trainees were able to transfer what they learned during the training to other areas, which establishes the external validity of the training method.

Table 3: Classroom Performance of Control and Experimental College Groups

Group	Mean Score (over 100)	Standard Deviation
Control	34	12.52
Experimental	60	15.18

t = 22.41

p = 0.0001

Discussion

Our results show that the proposed training method can improve significantly students' problem solving ability in physics. The method had two dimensions; one was concerned with student-teacher interaction that was dialectic; the other, which the content of instruction that prescribed explicitly how to build physico-mathematical models for solving problems. However, and due to uncontrollable limitations, it was not possible to make a multidimensional assessment of the training method. Separate and interactive effects of the dialectic approach and the modeling prescription were not evaluated.

Heller and Reif (1984) have shown that prescriptive, passive (non dialectic) training can improve students ability to solve specific problems. He had shown elsewhere that the dialectic active approach give better results, especially that it helps students resolve incompatibilities between their own knowledge state and scientific one, and transfer the prescribed procedures into other fields of knowledge (Halloun and Hestnes 1987).

Chi et al (1981) found that physicists are better problem solvers than students, mainly because of the special care they give to the description stage in the construction of the physics model, a stage that is not cared for by students. Heller and Reif (1984) had also shown that their subject's improvement in problem solving was mainly due to their ability to achieve the description of problems in a manner similar to the one proposed in our prescription. Our results show that this was also the case with our trainees (Tables 1 and 2).

Those trainees who were able to accomplish correctly the description stage but were not able to achieve correctly the solution of a particular problem did so because of deficiencies in their mathematical skills, mainly those concerned with vectorial operations. Such an impediment appeared mostly with high school trainees who were unable to break up force-vectors

into appropriate components. This helps explain why college trainees overall performance was significantly better improved than high school trainees, in addition to the fact that the pretraining competence of the latter was very poor.

Hestenes (1987) argued that model development strategies are common to all sciences. He even developed a modeling strategy that is suitable for solving any scientific problem. The prescription we proposed in this article was partly based on Hestenes model. The fact that college trainees' performance in the various parts of their formal physics course significantly better than that of the control group (Table 3) proves that, once students develop such modeling strategy in one specific scientific field, they can, with appropriate training, transfer such procedural knowledge to other fields.

The dialectic approach followed in this research is suitable for small classes. For classes of over about 40 students, the student-teacher interaction cannot be fairly controlled within the classroom. Students preinstructional knowledge that is incompatible with scientific theory and practice still can be exposed by the teacher in large classes, and the deficiencies of such knowledge pointed out, in the course of presenting the proposed prescription. An effort is being made currently in this direction by the author and some of his colleagues. Although it is still early to advocate for the issue, it seems that major developments can be made in this direction.

Abstract

A dialectic teaching method is proposed that prescribes how to develop appropriate physico-mathematical models for solving physics problems. Secondary School and College students who were trained following this method showed a significant improvement in solving Mechanics problems. Furthermore, the achievement in a university physics course of college trainees was significantly better than that of a control group.

Résumé

Une méthode d'enseignement dialectique est proposée suivant laquelle la résolution de problèmes de physique se fait par le développement de modèles physico-mathématiques appropriés. La méthode a été appliquée dans l'enseignement d'élèves du cycle secondaire et d'étudiants universitaires. La capacité des enseignants à résoudre des problèmes de mécanique a été considérablement améliorée. En plus, les étudiants ainsi formés ont

accompli un cours universitaire de physique dans lequel ils étaient inscrits avec une moyenne considérablement supérieure à celle d'un groupe-témoin.

REFERENCES

- ARONS, A. 1981, Thinking, Reasoning and Understanding. *The Physics Teacher*, vol. 19, pp 166-172.
- CHAMPAGNE, A. B. Klopfer, L. E. and Anderson, J. H. 1980, Factors Influencing the Learning of Classical Mechanics. *American Journal Physics*, vol. 48, pp. 1074-1079.
- CHI, M, FELTOVICH, P. J. and GLASER, R. 1981, Categorization and Representation of physics Problems by Experts and Novices. *Cognitive Science*, vol. 5, pp. 121-152.
- CLEMENT, J. 1982, Students' preconceptions in Introductory Mechanics. *American Journal of Physics*, vol. 50, pp. 66-71.
- CLEMENT, J., LOCKHEAD, J. and MONK, G. S. 1981, Translational Difficulties in Learning Mathematics. *American Mathematical Monthly*, Vol. 88, pp. 286-290.
- GAMBLE, R. 1986, Simple Equations in Physics. *European Journal of Science Education*, vol. 8, pp. 27-37.
- HALLOUN, I. A. 1988, Compétence en Mathématiques des Étudiants de Physique. *Lebanese Journal of Science and Mathematics Education*, vol. 1, pp. 22-25.
- HALLOUN, I. A. and HESTENES, D. 1987, Modeling Instruction in Mechanics. *American Journal of Physics*, vol. 55, pp. 455-462.
- HALLOUN, I. A. 1986, Le Réalisme Naïf et l'Apprentissage de la Physique. *Recherches Pédagogiques*, vol. 17, pp. 31-53.
- HALLOUN, I. A. and HESTENES, D. 1985, Common Sense Concepts about Motion. *American Journal of Physics*, vol. 53, pp. 1056-1065.
- HALLOUN, I. A. In Press, Schematic Structure of Physics Concepts.
- HELLER, J. I. and REIF, F. 1984, Prescribing Effective Human Problem Solving Processes: Problem Description in Physics. *Cognition and Instructions*, vol. 1, pp. 177-216.
- HESTENES, D. 1987, Toward a Modeling Theory of Physics Instruction. *American Journal of Physics*, vol. 55, pp. 440-454.
- KUHN, T. S. *The Structure of Scientific Revolutions*, 2nd ed. (University of Chicago Press, Chicago).
- LARKIN, J. H. McDERMOTT, J. SIMON, D. P. and SIMON, H. A. 1980, Expert and Novice Performance in Solving physics Problems. *Science*, Vol. 208, pp 1335-1342.
- McDERMOTT, L. C. 1984, Research on Conceptual Understanding in Mechanics. *Physics Today*, vol. 37, N° 7, pp. 24-32.
- POSNER, G. J. and GERTZOG, W. A. 1982, the Clinical Interview and the Measurement of Conceptual Change. *Science Education*, vol. 66, pp. 195-209.
- REIF, F. 1986 Scientific Approach to Science Education. *Physics Today*, vol. 39, N° 11, pp. 48-54.
- REIF, F. and HELLER, J. I. 1982, Knowledge Structure and Problem Solving in Physics. *Education Psychologist*, vol. 17, pp. 102-127.
- STRAND, J. 1986, On Some New Trends in Physics teaching. *European Journal of Physics*, vol. 7, pp. 11-16.