Schematic Modeling for Meaningful Learning of Physics

Ibrahim Halloun
Department of Physics & Astronomy, Arizona State University
Box 871504, Tempe AZ 85287-1504
On leave from Lebanese University

Abstract

Schematic modeling is presented as an epistemological framework for physics instruction. According to schematic modeling, models comprise the content core of scientific knowledge, and modeling is a major process for constructing and employing this knowledge. A model is defined by its composition and structure, and situated in a theory by its domain and organization. Modeling involves model selection, construction, validation, analysis and deployment. Two groups of Lebanese high school and college students participated in problem solving tutorials that followed a schematic modeling approach. Both groups improved significantly in problem solving performance, and course achievement of students in the college group was significantly better than that of their control peers.

Appeared in
Journal of Research in Science Teaching
1996, Vol. 33, Iss. 9

For decades, educators have been complaining that a high school or a college student often "passes [her or his physics] tests frequently alas, with very little comprehension of what [she or] he has been doing" (Swann, 1950). Recent educational research has consistently shown major deficiencies that persist after instruction both in the structure of students' knowledge of physics and in their problem solving skills. In this article, a *schematic modeling* approach is proposed to help students learn physics in a meaningful way and resolve those deficiencies. An experiment for assessing the approach is reported.

Research shows that high school and college students bring to their physics courses a rich array of *folk conceptions* about the physical world that are incompatible with physics theory. After completing introductory physics courses students often: (a) hold still to their folk conceptions (Hake, 1994; Halloun and Hestenes, 1985; Hestenes, Wells, & Swackhamer, 1992), and (b) continue to believe that physics consists mostly of mathematical symbols and formulas (Halloun, 1995a; Hammer, 1989 & 1994; Redish, 1994a; Reif & Larkin, 1991). Moreover, their ideas about physics remain disconnected, incoherent and inconsistent (Halloun & Hestenes, 1985; Hammer, 1994; McDermott, 1993; Novak, 1987 & 1994; Redish, 1994a; Reif & Allen, 1992; Reif & Larkin, 1991).

High school and college students often attempt to solve physics problems: (a) by trial and error, (b) backwards from a numerical answer provided in a textbook, or (c) by invoking a solution presented in class to a problem that they wrongly assume to be similar to the one they are working on (Arons, 1981; Halloun, 1995a; McDermott, 1993; Novak, 1987 & 1994; Reif & Larkin, 1991; Strnad, 1986). They tend to view solving a physics problem mainly as a task for selecting mathematical formulas to relate variables in the problem (Halloun, 1995a; Hammer, 1994).

Consequently, physics instruction suffers from: (a) *low efficacy*, in the sense that students who are diagnosed before instruction as average or low competence students remain at that level after instruction, (b) *short term retention*, in the sense that even the best students forget most of what they learn shortly after completing a physics course (Tobias, 1990), and (c) *high attrition rates*, especially among students initially diagnosed as of low competence (Halloun & Hestenes, 1987; Tobias, 1990).

Some physics educators have argued that students' evolution from folk realism to scientific realism can take place, at any level, only when the structure of physics theory and physicists' mental processes are presented explicitly (Eylon & Reif, 1984; McDermott, 1993; Mestre, Dufresne, Gerace, Hardiman, & Touger, 1993). This evolution may best be realized in model-based instruction. Some findings at the college level are worth noting in this regard:

- Students can learn meaningfully the content of scientific knowledge when it is presented in the form of models (Clement, 1989; White, 1993). This facilitates the development of scientific inquiry skills, especially critical thinking (Clement, 1989; Stewart, Hafner, Johnson, & Finkel, 1992; White, 1993). Students' scientific discourse improves significantly, in particular when they are asked to defend the validity of their models (Stewart et al., 1992; White & Frederiksen, 1990).
- Physics students engaged in model-based instruction are far more successful than their peers in resolving incompatibilities between their folk conceptions and physics theory (Halloun & Hestenes, 1987; Wells, Hestenes, & Swackhamer, 1995; White & Frederiksen, 1990).
- Modeling skills are generic. Physics students can successfully transfer modeling skills that they develop in specific situations into novel situations, within and outside the domain of instruction (Clement, 1989; Halloun & Hestenes, 1987; White, 1993; White & Frederiksen, 1990).
- Physics students who are initially of average or low competence benefit the most from model-based instruction (Halloun & Hestenes, 1987; White, 1993).

Aside from the seminal works of Hestenes (1987, 1992, 1995), there is no comprehensive theory for the use of models in physics instruction. All the research cited above emphasizes the role of models in physics and instruction, but it shows no clear consensus as to what a model is or how to go about constructing and employing models of physics systematically.

This paper presents *Schematic Modeling* as an epistemological framework for physics instruction. This framework of instruction is founded on two tenets:

- ***Models*** *occupy the content core of physics (or any science for that matter). A model in physics represents a set of physical systems in some respects , and serves well-defined purposes .*
- ***Modeling*** *is a systematic activity for developing and applying scientific knowledge in physics (or any science).*

Consequently, the pedagogical expectation is that:

> *by learning how to structure the content of physics theory around* ***models****, and how to solve problems by* ***modeling****, students will reach a* ***meaningful understanding*** *of physics which resolves the deficiencies discussed above.*

This expectation was partially tested in a limited experiment involving two samples of Lebanese high school and college students. Participants were taught to build a few models of Newtonian mechanics and employ them in a modeling process for solving textbook problems. The outcomes of this experiment are reported and discussed following a presentation of schematic modeling which is the major concern in this article.

Schematic Modeling

Schematic modeling is an evolving epistemological theory grounded in cognitive research. It holds that models are major components of any person's knowledge, and that modeling is a major cognitive process for constructing and employing knowledge in the real world. Three of the most fundamental tenets of schematic modeling are:

1. "We build mental models that represent significant aspects of our physical and social world, and we manipulate elements of those models when we think, plan, and try to explain events of that world" (Bower & Morrow, 1990).
2. "Our view of the world is causally dependent both on the way the world is and on the way we are. There is an obvious but important corollary: all our knowledge of the world depends on our ability to construct models of it" (Johnson-Laird, 1983).
3. *Mental models* are internal to a person's mind. They are *tacit,* and cannot be explored directly. However, they can be explored indirectly via *conceptual models* which a person *communicates* with others verbally, symbolically or pictorially (and/or via *physical models,* which are material artifacts). Conceptual models that we communicate in our everyday life are often subjective, idiosyncratic, and not coherently structured. With appropriate instruction, these models can become relatively objective and coherently structured (Gentner & Stevens, 1983; Giere, 1992; Hestenes, 1995; Nersessian, 1995; Redish, 1994). Such an evolution is best reached in science and mathematics where models occupy a pivotal role (Bronowsky, 1953; Casti, 1989; Giere, 1988; Hesse, 1970; Leatherdale, 1974).

Galileo (1564-1642) set the foundations for a modern epistemology of science whereby real world systems are studied indirectly through abstract models. He showed us how to build *reduced, idealized* models (like particle models) of physical systems, conduct *thought experiments* with such models, and consequently infer valid descriptions, explanations and predictions about physical systems. Science has since evolved more and more through model development. However, physics textbooks fail to present the fact that models occupy the content core of scientific theories, and that modeling is a major process –if not the major one– for developing and employing such theories.

Recently, some science and mathematics educators have tried to correct this deficiency by explicitly advocating modeling in their disciplines (AAAS, 1990 & 1993; NRC, 1996), from biology (Hafner, 1992; Hafner & Stewart; 1995; Smith, 1992; Stewart et al., 1991 & 1992) to physics (Clement, 1989; Halloun & Hestenes, 1987; Hestenes, 1987, 1992 & 1995; Nersessian, 1995; Pollak, 1994; Raghavan & Glaser, 1995; Redish, 1994b; Wells et al., 1995; White, 1993; White & Frederiksen, 1990), as well as in mathematics (AMATYC, 1995; Casti, 1989; Edwards & Hamson, 1989; Mac Lane, 1988; Steen, 1990; Swetz & Hartzler, 1991). It has even been argued that unless students are "introduced to the game that professional scientists play called 'creating and shooting down models' [we do not] let them in on the game of 'being' a scientist" (Pollak, 1994).

What is a Scientific Model?

When studying physical systems, physicists concentrate on a *limited number* of features that they deem *primary* in the structure and/or behavior of these systems, i.e., pertinent to the purpose of their study. They build a *conceptual model* (e.g., a mathematical one) and/or a *physical* one (a material artifact) that bears only these primary features. They analyze the model thus built, and then they draw inferences about the physical systems represented by the model. The entire process is usually governed by some theory of physics. This article concentrates on conceptual models because they are: (a) the blueprints of physical models, and (b) the ones that are mostly used in physics instruction.

In contrast to the relatively *idiosyncratic* conceptual models of non-scientists, scientists, at least those in the same discipline, share the *same* conceptual models about specific systems and phenomena in the natural world. Non-scientists' models will hereafter be referred to as *folk models*, and those of scientists as *scientific models*. Scientific models are *schematic* in the sense that, like other scientific *schemas* (concepts, laws and other conceptual structures shared by scientists), they are: (a) *reduced* to a limited number of *primary* features that are almost independent of the idiosyncrasies of individual scientists, and (b) developed and applied following *generic* modeling *schemes*, i.e., systematic plans of interaction with the real world that enable one to construct and employ not only some models of interest but all sorts of new schemas.

Knowledge that one needs to develop for a comprehensive understanding of a scientific conceptual model (or model hereafter, for short) can be comprised in four dimensions: domain, composition, structure, and organization. Composition and structure "define" the model. Domain and organization situate the model in the theory to which it belongs. The four dimensions are outlined below. Details can be found elsewhere (Halloun, 1995b; Hestenes, 1995).

1. Domain

The domain of a model consists of a set of physical systems (called the *referents* of the model) which share common structural and/or behavioral features that the model can help us *describe*, *explain*, and/or *predict* in some respects, and to a certain degree of approximation and precision. The model could subsequently allow us to *control* these referents, and *design* new ones.

Depending on: (a) what features are modeled, and (b) the desired approximation and precision, a given physical system may be represented by different models that could belong to the same theory or to different ones. For example, consider a physical object in motion. Depending on the speed of the object, and the extent to which its mass and shape are approximated as constant, its motion can be studied using models from Newtonian mechanics or from Relativity. Two kinds of models are commonly distinguished in Newtonian mechanics: particle and rigid bodies (details below). A particle model can be used to describe (kinematics), explain (dynamics) or predict the motion of the object if it is in simple translation; but such a model would be inadequate, or at least insufficient if the object was also spinning, in which case a rigid body model would be needed.

2. Composition

When modeling a physical situation, scientists tend first to group objects involved into finite systems. Each system would include one or more *physical entities* that exhibit specific *properties* of interest and that *interact* with each other, as well as with some other physical entities outside the delineated system. Physical entities inside and outside the system can be represented in the corresponding model by *conceptual entities* that belong to the *content* and the *environment* of the model respectively, and that are characterized by appropriate *descriptors*. Content and environment, along with respective object and interaction descriptors, make up the *composition* of a model.

2.a Content

The content of a model consists of *objects* (more specifically object-concepts or conceptual objects) representing physical objects inside its referents. A model (or a physical system) can be: (a) *simple,* if consisting of only one object, or (b) *composite,* if consisting of more than one object. For example, a particle model of Newtonian mechanics is a simple model consisting of a single object called a “particle”, and commonly depicted by a geometric point in a given coordinate system. Many-particle models, like an ideal gas model, are composite models. The boundaries of a physical system can always be conveniently set so that it be represented either by a simple model or a composite one. Simple models though are often more convenient.

2.b Environment

The environment of a model consists of *agents* (object-concepts) representing physical entities outside its referents that are interacting with entities inside. An entity outside a physical system is represented by an agent in the corresponding model only if it affects significantly what is being studied. For example, every object on Earth is physically attracted by the Moon and all other celestial objects. However, when studying the motion of terrestrial objects in Newtonian theory, gravitational interaction is considered only with Earth, that with the Moon and other celestial objects being practically negligible. In modeling terminology, Earth is the only celestial object that is represented by an agent in the environment of models referring to terrestrial objects.

2.c Object descriptors

A *descriptor* is a property-concept or a *conceptual* property (a *variable*, in a mathematical model) that represents a specific physical property of entities inside or outside the referents of a model (Halloun, 1996). An *object descriptor* is a characteristic feature of an object in the content of a model. It is a *conceptual* property of the object that represents in some respects a *physical intrinsic* or *state* property of entities inside a physical system represented by the model.

An object descriptor can thus be an intrinsic descriptor or a state descriptor. An *intrinsic* descriptor, or *parameter,* represents a physical property that is assumed to be *constant* (e.g., mass or moment of inertia of a rigid object). A *state* descriptor, or *variable* in the narrow sense, represents a physical property that can vary in time (e.g., position and kinetic energy of an object in motion).

Like an object, an agent can also have its intrinsic and state descriptors. The utility of these descriptors though is often limited to the quantification of the interaction between the agent and respective object(s) as explained below.

2.d Interaction descriptors

An interaction descriptor is a mutual feature shared by an object and an agent in a model. It represents a property of *physical interaction* between an entity inside a referent of the model and one outside (e.g., force and potential energy). It is often convenient to build a model in such a way that we can ignore interactions among agents and the effect of any interaction on an agent. In such a case, only the action of an agent on an object needs to be accounted for (Figure 1). For example, in Newtonian models, only the force imparted *by* an agent *on* an object is considered; the one exerted by the object on the agent is ignored (Halloun, 1996).

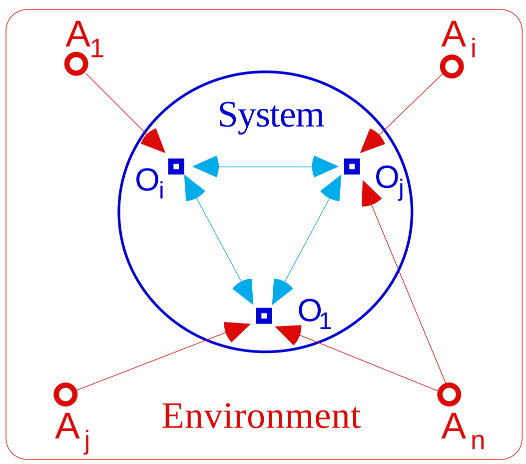


Figure 1: Partial diagrammatic representation of the composition of a model.
Note that: (a) interactions are depicted with two-way arrows between two objects (O's), and one-way arrows between agents (A's) and objects, and that (b) no interaction is shown between an object and itself, or among agents.

A model is not isomorphic with, or a mirror image of, any of its referents. Not every entity of a physical system needs to, or actually can, be represented in a model representing it. The same is true for properties of the referent. However, every object in a model must correspond to at least one entity inside its referent(s), and every agent, at least one entity outside. Similarly, every descriptor in a model must correspond to a specific physical property of its referent(s). One though can always build conceptual models that do not map entirely on the set of its referents; but these models would not be scientific then.

3. Structure

The structure of a model consists of *relationships* between descriptors that represent to a certain degree how physical properties of the model's referent(s) affect each other. Various relationships comprise three types of structure: geometric, interactive and behavioral.

Geometric structure refers to the spatial configuration of objects and agents. Such structure is often expressed in terms of the *position* of individual objects and agents in a given reference frame, and/or of *non-temporal* relationships between the relative positions of the various parts of individual entities.

Interactive structure refers to *non-temporal* relationships expressed in *interaction laws* between an interaction descriptor and object descriptors of the respective object and agent (e.g., Newton's law of Universal Gravitation, Coulomb's law of electrostatic interaction and Hooke's law).

Geometric and interactive structures are called *internal* when they relate descriptors of various objects in the content of a model to each other but not to those of agents in its environment. The structures are called *external* in the opposite case. Simple models have no internal structure, whence the convenience of using them whenever possible.

Behavioral structure refers to *spatio-temporal* relationships that describe or explain the behavior under specific conditions of individual objects in the content of a model. These relationships are expressed in two types of laws: state laws and causal laws.

State laws express relationships between *object properties* of a single object, and *describe* the *change* of state of a particular object, i.e. the variation of its state descriptors (e.g., Newton's first law and kinematical laws of motion often referred to in textbooks as *equations of motion*).

Causal laws express relationships between an interaction property and state properties of an object, and *explain the change* of state of an object (e.g., Newton's laws of dynamics and conservation laws).

A model is called *descriptive* when it does not have an interactive structure *and* when its behavioral structure is expressed only in terms of state laws but not causal laws. The model is called *explanatory* when it does have an interactive structure and/or when its behavioral structure is expressed only in terms of causal laws but not state laws. A comprehensive model is one that is both descriptive and explanatory, i.e. that includes all types of structure. A system may be represented by anyone of the three types of model depending on the purpose of study.

4. Organization

Models belonging to the same theory can be classified into groups and subgroups (or families) of models following convenient criteria. Each group includes a special *family* of models called *basic* models. A basic model is often a simple but comprehensive model that describes and explains an elementary physical phenomenon.

Two of the most fundamental model groups in Newtonian mechanics are particle models and rigid body models. *Particle* models refer to physical objects the internal structure of which can be ignored when they are in *translation* without rotation or precession, in a specific reference system. The content of a *basic* particle model consists of a single, dimensionless object, a particle, undergoing translation under a specific type of force. *Rigid body* models refer to physical objects the internal structure of which cannot be ignored when they are in *translation* and/or *rotation* and/or *precession*. The content of a *basic* rigid body model consists of one solid of a regular geometric shape rotating (precession ignored) about a specific axis under a specific type of torque. Figure 2 shows the two families of basic models in Newtonian Theory.

Basic models are indispensable: (a) for meaningful understanding of individual concepts and principles in a given scientific theory, and of modeling rules, and (b) for developing more complex models. Mastery of basic models is thus essential for evolving from a folk conceptual world into the scientific world (Giere, 1994; Wells et al., 1995).

In addition to classification criteria, every theory contains *organization laws and rules* that specify: (a) how models within a given family relate to each other and to those in other families (e.g., how every particle model relates to other particle models in the Newtonian theory and to rigid body models), and, consequently, (b) how to combine different models for studying physical situations that are outside the comprehensive domain of available models. For

Basic Particle Models

Free Particle

This model refers to physical objects subject to zero net force ($\sum \mathbf{F}_i = 0$) in linear translation with constant velocity or at rest.

Uniformly Accelerating Particle

This model refers to physical objects subject to a net constant force ($\sum \mathbf{F}_i$ = constant), hence moving with constant acceleration in a linear or parabolic path.

Harmonically Oscillating Particle

This model (often called simple harmonic oscillator) refers to physical objects subject to a net force that is proportional to their displacement from a center of force ($\sum \mathbf{F}_i \quad \Delta \mathbf{r}$), hence undergoing simple harmonic motion.

Uniformly Circling Particle

This model refers to physical objects subject to a net centripetal force ($\sum_{radial} \mathbf{F}_i \quad 1/r^2$) of constant magnitude, hence undergoing a uniform circular motion.

Basic Rigid Body Models*

Freely Rotating Rigid Body

This model refers to physical objects subject to zero net torque about a specific axis ($\sum \tau_i = 0$), hence rotating with constant angular velocity about this axis or at rest.

Uniformly Accelerated Rotating Rigid Body

This model refers to physical objects subject to a net constant torque about a specific axis ($\sum \tau_i$ = constant), hence rotating with constant angular acceleration about this axis.

Figure 2: Two families of basic models in Newtonian Theory.

* Euler's laws (which are sometimes wrongly referred to as "Newton's laws of rotation") govern the rotation of rigid body models. Euler's laws follow the Newtonian philosophy. In this respect, the family of rigid body models is here considered part of the "Newtonian" theory. One, though, could still refer to "Newtonian Theory" in the narrower sense, and restrict it to particle models.

example, appropriate organization rules in the Newtonian theory would tell us how to combine the uniformly accelerating particle model with the uniformly circling particle model of Figure 2 in an *emergent* model that is suitable for describing, explaining, and/or predicting the behavior of an object that undergoes a uniformly accelerated, circular translation. If the object were also rotating about a specific axis, such rules would further tell us how to combine the emergent model thus constructed with an appropriate rigid body model in a new more suitable model.

A word of caution is due at this point. The four dimensions presented above are meant for teachers and not students to serve as templates or guides for planning and evaluating instruction. Students need to develop knowledge contained in all dimensions in order to develop comprehensive and meaningful understanding of a model. However, teachers need not –and actually should not, at the introductory level– present the dimensions in the way presented above to students. Instead, every teacher should guide students to develop contained information by asking specific questions tailored to the particular knowledge state of respective students. The appendix associated with the next section offers some suggestions on how to do so in the case of problem solving.

The Modeling Process

Figure 3 shows a generic modeling process that can be systematically applied in the context of a convenient theory for building new models, refining them and/or employing them in specific situations (Hestenes, 1995). These situations may appear in the real world, laboratory experiments, or textbook problems.

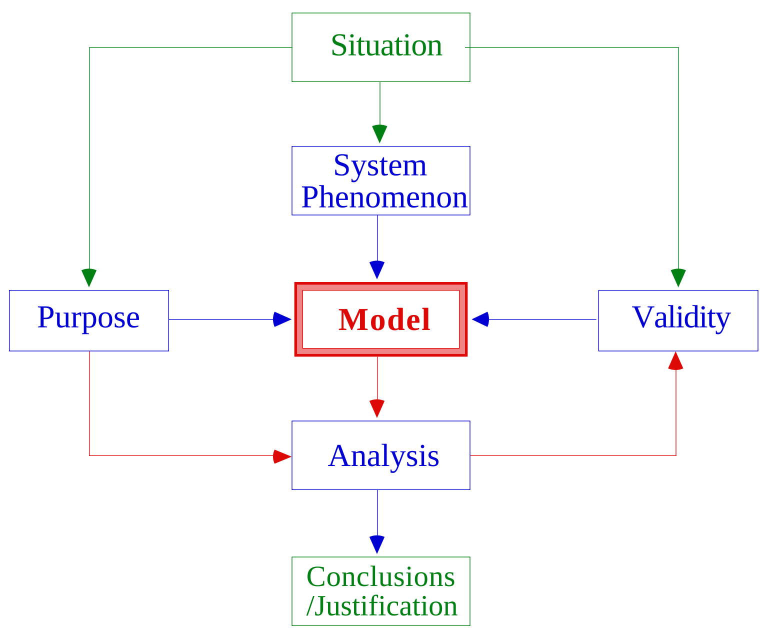


Figure 3: Schematic representation of the modeling process (Hestenes, 1995).

The first stage in the modeling process consists of *identifying* and describing the *composition* of each *physical system* in the situation, and the respective *phenomenon.* Then, or concurrently, the modeling *purpose* will be identified (e.g., goals set in a textbook problem), as well as the *validity* of the expected outcomes (including approximation and precision limits). Following these steps which are critical for choosing the appropriate *theory* in the context of which modeling would proceed, an appropriate *model* is *selected* (whether physical or conceptual) and *constructed*. The model is then *processed* and *analyzed*, while it is being continuously *validated*. Following analysis, appropriate *conclusions* are inferred about the system in question, as well as about other referents of the model, and outcomes are *justified* in function of the modeling purpose and the required validity (Halloun, 1995b; Hestenes, 1995).

In physics instruction, the modeling process can be used in experimental activities as well as in learning textbook material and solving respective problems. In the following, I illustrate how this schematic process applies in solving textbook problems. My choice of problem solving is simply due to the fact that physics instruction relies heavily on it for teaching and, especially, for assessing student understanding of physics in paper-and-pencil exams.

Modeling for Solving Paradigm Problems

Some physics textbook problems that I call *paradigm problems* are exceptionally helpful for learning the modeling process in non-empirical settings. Paradigm problems have special features ranging from avoiding straightforward numerical applications of formulas to including open-ended questions that allow students to reflect on their folk conceptions about physical systems (Halloun, 1996).

Modeling for solving paradigm problems goes in five stages: selection, construction, validation, analysis, and deployment. The process is not hierarchical; the middle three stages overlap, and some of their steps can often be conducted concurrently. In each stage, students ask themselves specific questions, and answer them systematically. Typical questions are presented in the appendix for solving Newtonian mechanics problems. The reader will notice that some of the questions are generic and could apply to any problem, while others are specific to Newtonian mechanics but could easily be deployable into other fields. One does not always need to follow the modeling process below to the letter, and may modify it by deleting or adding specific items in order to adapt it to special needs.

Teachers should by no means prescribe such a modeling process passively to their students; they should guide them to develop it gradually as they go about solving paradigm problems. In the appendix, I outline some of the questions that students should learn to ask themselves, and answer on their own, in modeling for solving paradigm problems. Teachers should help students realize the importance of every question, and appreciate the need for such a comprehensive process for successful problem solving. Such an endeavor is best achieved in an *interactive*, *dialectical* and *motivating* classroom environment, whereby students collaborate with each other in classroom activities that stimulate their interest, and that allow them to reflect back on their folk models and resolve any incompatibilities with scientific models (Halloun, 1996).

1. Model Selection

Solving textbook problems often involves basic models and/or emergent models which are combinations of specific basic models. Hence, the respective modeling process always starts by selecting an appropriate model(s) from a repertoire of familiar models in a specific theory (e.g., Figure 2). The selection is guided by the domain of each model, and governed by the modeling *purpose* and required *validity*, as explained above.

2. Model Construction

In this stage, students are guided to construct a *mathematical model* that helps them solve the problem. They construct (or reproduce) the composition and the structure of every selected model. These two components do not need to be specified exhaustively as presented above in every problem. Some problems may require purely *descriptive models* whereby the environment and respective interactions and causal laws need not be specified (e.g., problems of kinematics). Other problems may require purely *explanatory models* whereby only interactions need to be identified and expressed with appropriate interaction laws (e.g., statics problems), and/or causal laws. The best paradigm problems are those that require comprehensive models that are both descriptive and explanatory.

3. Model Validation

This stage can sometimes proceed along with model construction, especially with regard to internal consistency (Appendix). Validation includes different forms of assessment that provide students with opportunities to fulfill a major objective of science education: *critical thinking*. Various forms of assessment are listed in the appendix.

4. Model Analysis

Once a model is validated, at least through consistency (Appendix), analysis can proceed to fulfill the purpose for which it is being constructed. Model analysis in solving textbook problems consists primarily of processing the mathematical model, getting answers to the questions asked in the problems, and interpreting and justifying the answers.

5. Model Deployment

Once a model is analyzed and fully validated, implications can be inferred with respect to the original purpose, as well as with respect to other valid purposes. This helps students develop *transfer* skills. Model deployment includes:

- Using a given model to describe, explain, and/or predict new physical situations pertaining to the system(s) in the problem.
- Inferring implications for other referents of the model.
- Extrapolating the current model to build new ones.

Model deployment also includes *reflective* activities, whereby students examine and refine their current knowledge in terms of the new modeling experience (Appendix).

Method

The schematic modeling framework of instruction was partially tested in a limited experiment with two groups of Lebanese students. The experiment and its outcomes are described and discussed below. Because of practical constraints, controls on the experiment were less than ideal. Nevertheless, the outcomes show the potential of schematic modeling to significantly improve physics instruction, and provide ample grounds for further research.

Participants

Two groups of 87 Lebanese students participated in tutorials designed to help them construct some basic models of Newtonian mechanics, and employ them in solving paradigm problems. One group consisted of 59 high school students, the other of 28 college students.

High school subjects were enrolled in a First Secondary class (Grade 10), and had taken their first annual physics course. About half the course is devoted to the statics of rigid bodies, and covers applications of Newton's first law (for translational equilibrium) and Euler's first law (for rotational equilibrium). The rest of the course covers hydrostatics and heat. Participating subjects had failed their physics course and were required to pass a special remedial session during the following summer. During five two-hour periods of this session, students were taught to solve statics problems following a schematic modeling approach. Because of administrative constraints, all 59 students participated in the model-based tutorials, and there was no control group.

College students were enrolled in a calculus-based introductory physics course. The course runs for one semester, and covers translational and rotational motions in classical mechanics, waves and hydrodynamics. A total of 107 students were enrolled in the course, at the time of the experiment. After completing the part of the course pertaining to particle models, and before working on rigid bodies, a special tutorial session was advertised for students to learn the schematic modeling approach in particle mechanics. The session consisted of five two-hour meetings, and was attended by 28 students. The remaining 79 students did not receive any extracurricular tutorials. These students served as a control group in a limited part of the experiment as will be discussed below.

Previous research had shown that social factors, including gender, have no significant effect on Lebanese students' performance in physics (Halloun, 1986). Consequently, no such data were collected for participating students. In the college experiment, homogeneity of the experimental and control groups was verified before the tutorials using a standardized mechanics diagnostic test whose validity and reliability are well documented (Halloun & Hestenes, 1985; Halloun, 1986). Subjects in the control group and the experimental group averaged 52% and 51% respectively on this instrument.

Procedures

Both high school and college tutorials were conducted by the author who was not the teacher of either course in which participants were enrolled. High school tutorials pertained to the free particle model and the freely rotating rigid body model (Figure 2). College tutorials pertained to the entire family of basic particle models in Figure 2.

Tutorials involved solving *paradigm problems* carefully chosen from students' textbooks, following an interactive, dialectical approach (Halloun, 1996). In every two-hour meeting, two or three paradigm problems were discussed. A group of two students would be asked to present a solution to a specific problem, and engage in a discussion with their peers on the validity of their solution. The choice of group members was constrained by the ability of any two students to meet and work on assigned problems outside the classroom. The same assignment of two or three problems was given at the end of every meeting to all participating subjects. Presenters were not designated ahead of time; they were selected randomly at the time the group met for tutoring. A modeling process similar to the one presented above and in the appendix for solving paradigm problems was gradually developed during tutorials by the members of each experimental group, with the guidance of the author.

Special care was given to inventorying the *environment* of a model, because, like in previous studies, pretests in this experiment have shown that this is a critically weak component in students' problem solving skills. Subjects were taught explicitly how to use the *schematic* concept of force (Halloun, 1996). They learned how to identify *agents* acting on an *object*, how to specify the corresponding forces, and how to draw appropriate *force diagrams* (Figure 4).

Models used in high school tutorials were *explanatory*. Only Newton's and Euler's first laws were needed to explain the statics of a given object. No quantitative description of motion was involved. Models used in college tutorials were both *descriptive and explanatory*. The behavior of every object was described by appropriate *motion maps* showing geometric depictions of particle kinematics (Figure 5). Causal laws included Newton's laws and conservation laws. Special attention was given to relating velocity and acceleration vectors and matching the acceleration of a particle with the respective net force vector.

Internal validity of the tutorials was partially assessed using parallel pretests and posttests consisting of paradigm problems. *External validity* was partially assessed only with the college group in terms of overall achievement in the course of enrollment.

Results

A pretest consisting of two paradigm problems was administered to the high school group just before the tutorials. A similar posttest was administered at the end of the tutorials. Figure 6 depicts the situations in one pretest and one posttest problem. These particular situations were chosen because their solutions require both Newton's and Euler's laws. On the depicted pretest problem, students were asked to evaluate the tension in the central rope and the relative positions of the loads suspended from the horizontal rod. On the depicted posttest problem, students were asked to evaluate the pull required by the person as well as the force exerted by the wedge to hold the beam in the position shown. Neither the pretest nor the posttest problems were discussed during the tutorials.

As shown in Figure 6, only 5% of students provided a valid complete solution to the pretest problem, whereas 34% were capable of doing so on the posttest problem. Figure 6 also shows pretest and posttest performance on critical steps of the modeling process required for solving the assigned problems. One critical step consists of listing agents acting on the object in the problem (environment), followed by identifying and drawing respective forces (interaction description). The behavior of each object (at rest) needed to be explained using Newton's first law for translational equilibrium, and Euler's first law for rotational equilibrium. Successful model analysis depended critically on students' ability to decompose force vectors into appropriate components, and write corresponding equilibrium equations. A closer look at students' performance on the pretest and posttest revealed that successful accomplishment of each step depended on success with the immediately preceding step (Figure 6).

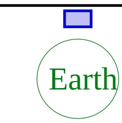
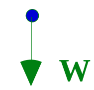
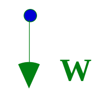
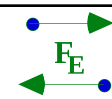
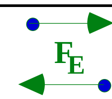
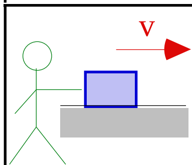
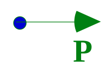
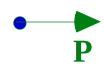
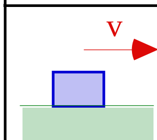
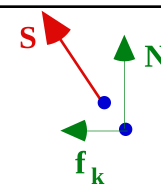
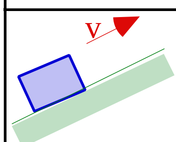
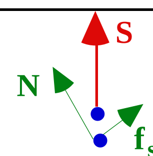
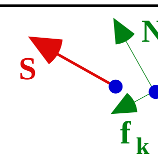
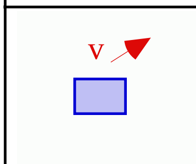
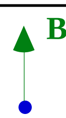
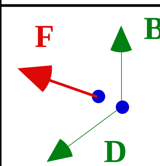
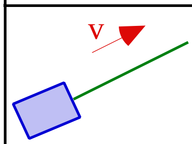
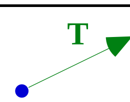
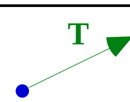
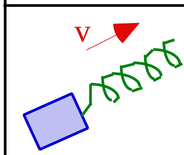
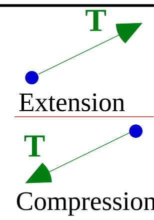
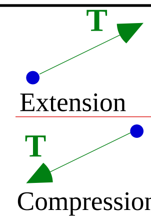
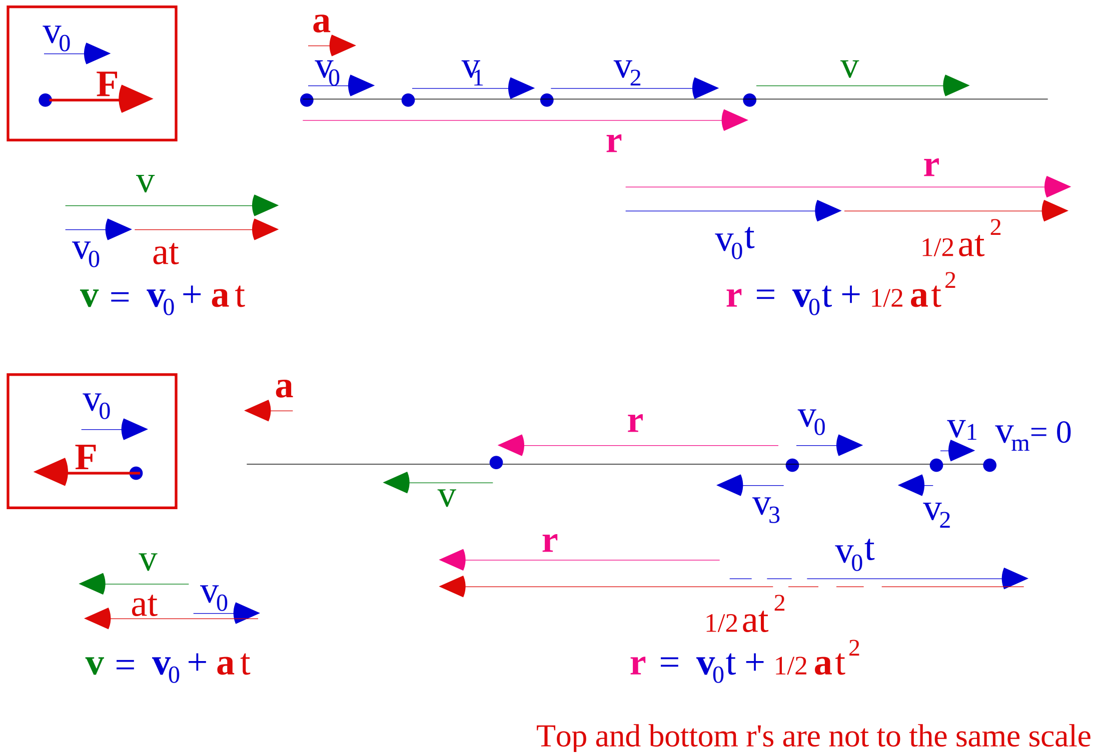
	Agent Name	Examples	**Force Name**	**Force Diagram** Rest	Motion
Long-range Interactions					
	Any physical object	Earth, Moon, Sun, other planets	Gravitational force or Weight **W**		
	Electric charge carriers	Electrons, protons, ions, etc.	Electrostatic force $\mathbf{F_E}$		
Contact Interactions					
	Direct mover	Human hand, one car directly pushing another car	Traction: Push, pull **P**		
	Horizontal Solid Support	Table, ground, road, shelf, board, human hand	Support force **S**. Components: Normal **N** & friction **f**		
	Inclined Solid Support	Table, ground, road, shelf, board, human hand	Support force **S**. Components: Normal **N** & friction **f**		
	Fluid	Air, water, other gas / liquids	Fluid force **F**. Components: Buoyancy **B** & drag **D**		
	Rigid Suspender	Rigid rope, string, rod, bar, or chain, human arm	Tension **T**		
	Elastic Suspender	Spring, Elastic rope, string, rod, bar, or chain	Restoring force **T**		

Figure 4: Sample force catalog for particle models.

Particle subject to a constant force F that is collinear with its initial velocity v_0



Particle subject to a constant force F that is not collinear with its initial velocity v_0

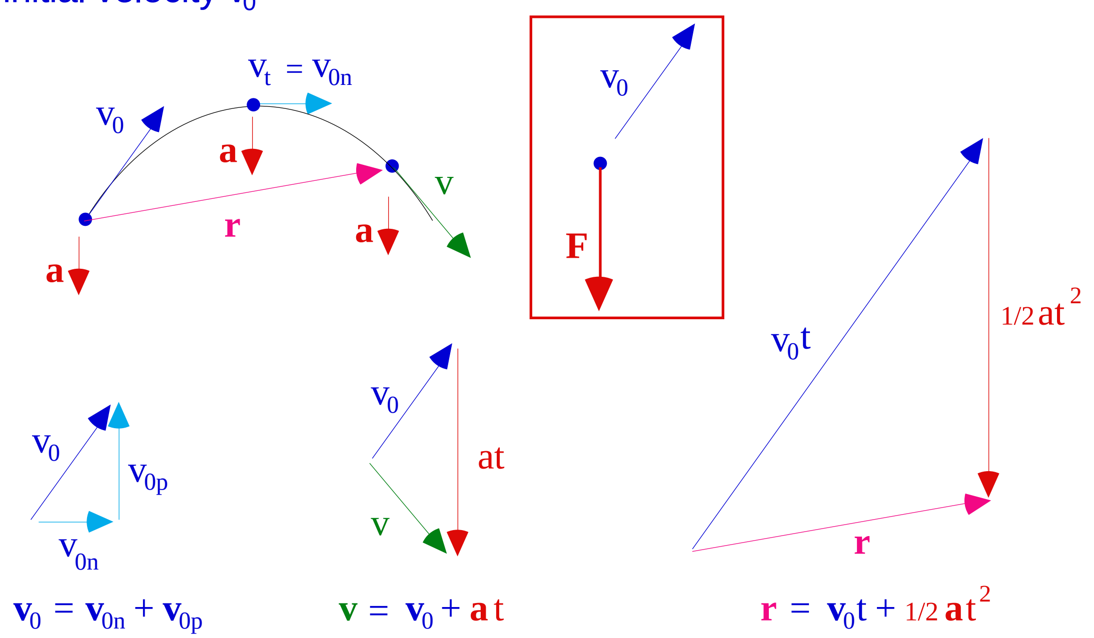


Figure 5: Motion maps of a constantly driven particle.

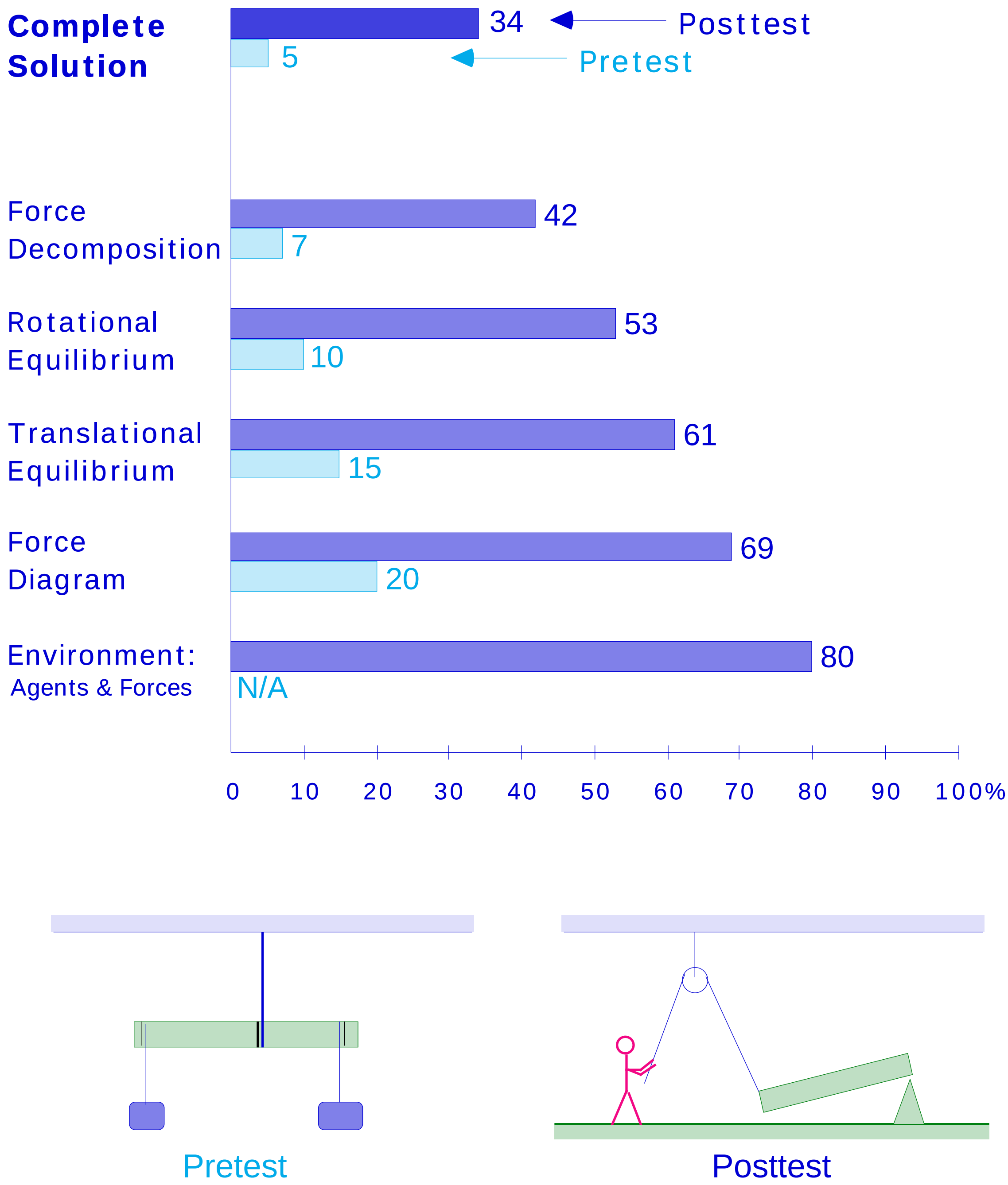
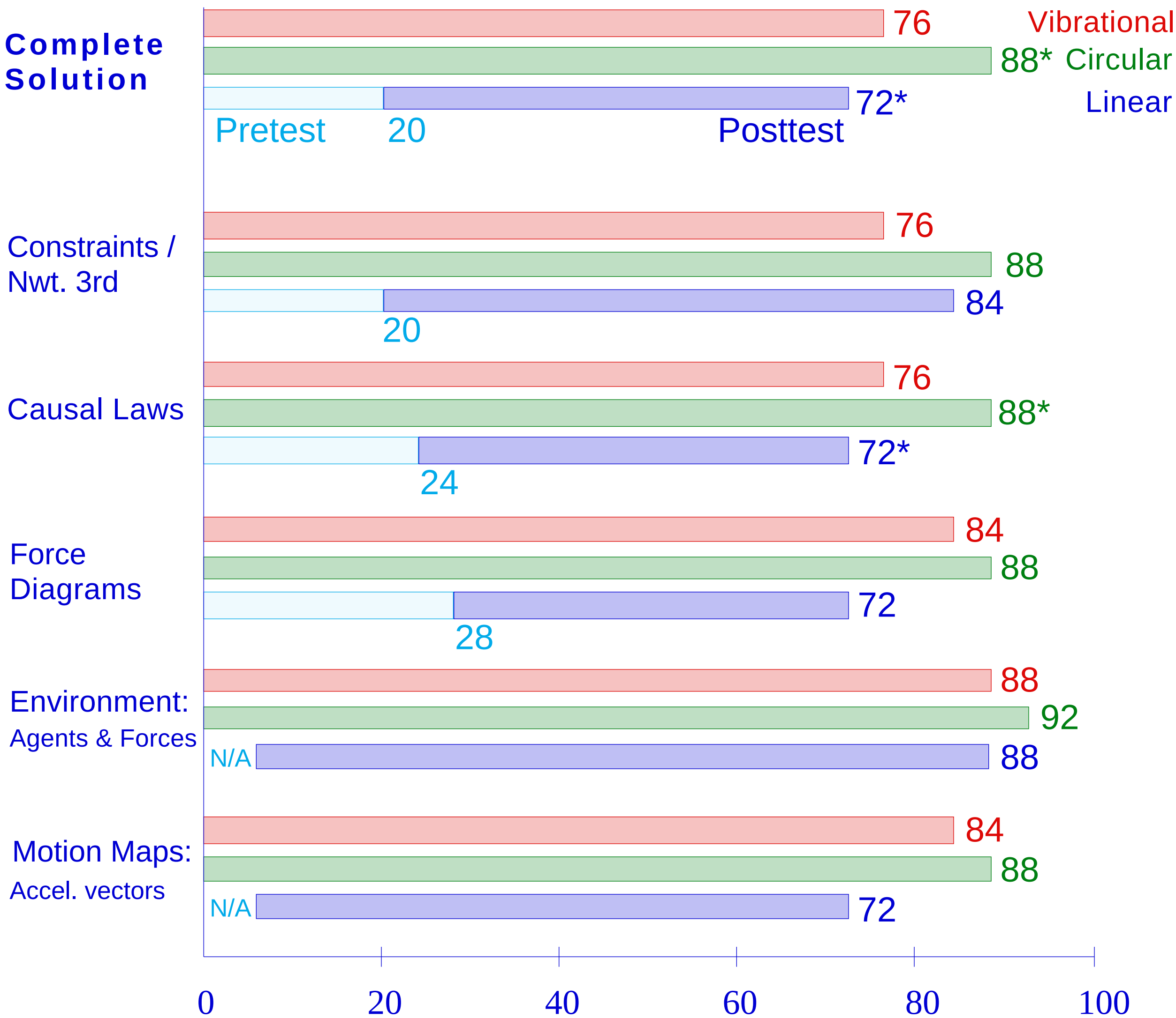


Figure 6: Problem solving performance of high school participants on parallel pretest (bottom light bars) and posttest (top dark bars) tasks.



* All students who did correctly the a & F vector diagrams completed tasks indicated with a star

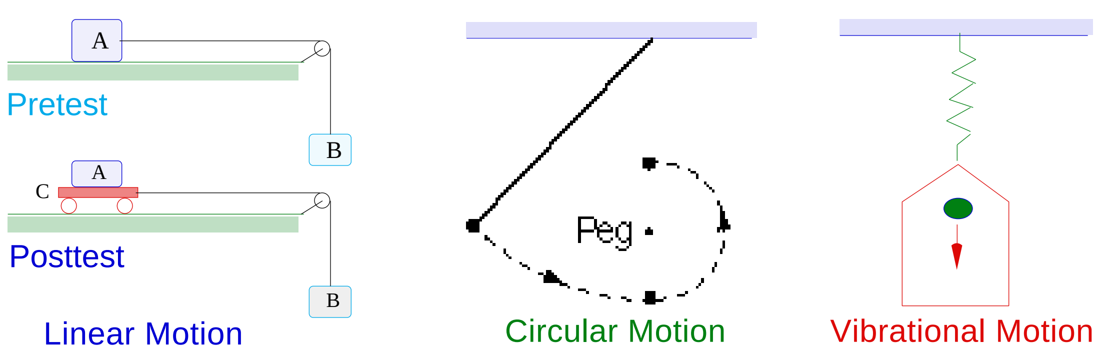


Figure 7: Problem solving performance of college participants on a pretest task (linear motion, light left portions of bottom bars) and three posttest tasks (vibrational, circular and linear motions respectively, from top to bottom).

On the posttest, nine high school participants (15%) did not list explicitly agents acting on the object, and three others (5%) committed mistakes in doing so. All these students failed to solve the problem correctly. Of the 47 students who were able to identify correctly all agents (Earth, the rope and the wedge), six students (13%) drew wrong force diagrams, and subsequently made consistent mistakes in completing the model's structure.

All students who could not write a correct translational equilibrium equation were also unable to write one for rotational equilibrium. All students who wrote a correct rotational equilibrium equation had done so for translational equilibrium. This shows that mastery of the *freely rotating rigid body* model depends on mastery of the *free particle* model. It is likely, therefore, that an understanding of the family of particle models is necessary for learning the family of rigid body models.

A similar pretest-posttest assessment of problem solving skills was conducted with the college experimental group. Figure 7 shows students' performance on three posttest problems with situations depicted in the bottom diagrams. The pretest consisted of two linear motion problems, one of which, depicted in Figure 7, was somewhat parallel to, but simpler than, the linear motion problem in the posttest. In the posttest problem, load A was supposed to skid to the left on top of cart C. The pretest did not include problems on circular motion or vibrational motion since there is no reason to believe that students could do any better on these problems.

Pretest–posttest comparison in Figure 7 shows a substantial improvement. Like the high school group, college participants' *ability to provide a valid complete solution to any problem hinged primarily on their ability to provide accurate object description (motion maps) and interaction description (force diagrams)*. A correct identification of a particle's environment was critical for the latter description. Explanation of the behavior of a particle required the use of Newton's second and third laws or of conservation laws.

Comparison of Figures 6 and 7 suggests that the schematic modeling instructional method was more successful with college participants than high school participants, especially in getting a complete solution to a problem. However, three major factors unrelated to the instructional method contribute to the difference.

First, the high school group consisted only of students who failed their first physics course, whereas the college group consisted of students of mixed competence. Of course, the college group also had a stronger physics background (Lebanese students are all required to take physics in high school).

Second, a close look at high school participants who failed to complete a desired solution revealed that these students were unable to decompose force vectors correctly, so they wrote wrong component equations. Hence, their failure was at least partly due to deficiencies in mathematical skills.

Third, the posttest problem given to high school participants was much harder than the respective pretest (Figure 6). When this is taken into account, and mathematical factors are eliminated by comparing gains on the bottom three steps of Figures 6 and 7, gains in the modeling of forces on a particle become quite comparable for both high school and college groups.

Taking all three points above into account, one can conclude that the gains of high school participants were no less significant than those of college participants . The results of both groups thus provide some corroboration for the *internal validity* of the schematic modeling tutorials, more specifically, its validity for significantly improving students' problem solving abilities within the domain of the tutorials.

Stronger corroboration would require comparison with control groups' performance on the same pretests and posttests. Such groups were not available at the time the experiment was conducted. Instead, the linear motion problem in the college posttest (Figure 7) was later given in a midterm exam in an introductory college physics course at a U.S. university. Comparative results are shown in Figure 8 with respect to the modeling steps appearing in the solutions of the 139 students who were enrolled in this course. In a previous study, physics and mathematics competence were assessed for students enrolled in the same courses at the same institutions implicated in Figure 8. U.S. students were then shown to be significantly better than their Lebanese counterparts (Halloun, 1986). Taking this into account, the results depicted in Figure 8 provide indirectly further support for the internal validity of model–based instruction.

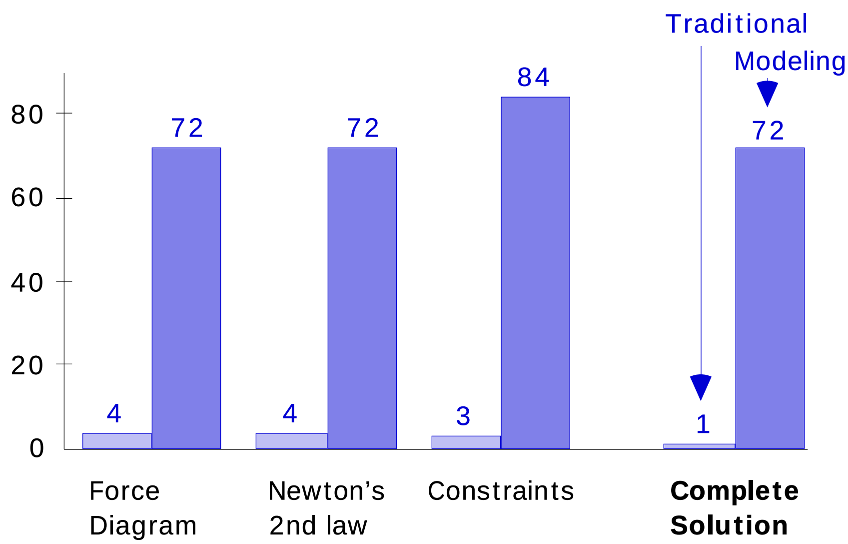


Figure 8: Performance on the linear motion problem (Figure 7) of college students in the modeling group (right bars) and in a traditional university course (left bars).

An assessment of the *external validity* of the tutorials was possible only for the college group. Final grades of all students were obtained for the college physics course in which they were enrolled. The mean scores of both experimental and control groups are compared in Figure 9 showing that students in the schematic modeling group averaged 26% better. A closer look at students' grades revealed that the difference was about the same on the part of the course pertaining to particle models as on the rest of the course (rigid body models, waves and hydrodynamics). This strongly indicates that students were capable of transferring modeling skills learned with some particle models during the tutorials into other domains. Participants transferred such skills on their own; course instruction was traditional and completely blind to the schematic modeling approach, and no follow up model–based instruction was provided.

Tutorials also had a long term impact on attrition. Three students (11%) in the modeling group withdrew from the course before the end of the semester, whereas thirty students (38%) of the control group withdrew (Figure 9).

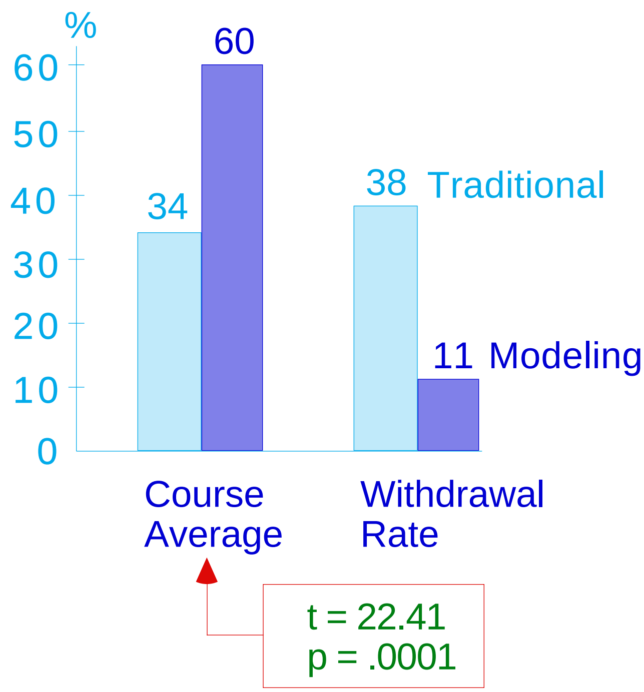


Figure 9: Course achievement of college students in the modeling group (Modeling, right ars) and the control group (Traditional, left bars).

Discussion

The results above provide some corroboration for the internal validity and the external validity of the tutorials. Subsequently, they imply that schematic modeling is a valid epistemological framework for physics instruction. Further research is required to assess individual components of the framework, mainly the importance of structuring the content of physics theory around models as characterized above, and of following the modeling process in various contexts of model construction, validation and deployment (including problem solving). Such a research plan is well beyond the scope of this paper, or of any single research project for that matter. However, the outcomes of the limited experiment reported in this article give clear indications to what makes a schematic modeling approach valid and successful.

Results shown in Figures 7 and 8 indicate that, at least in mechanics problems, a correct solution to a problem hinges on one's ability to identify correct agents in the environment of an object, and depict interactions with appropriate force diagrams. Motion maps depicting the kinematical state of an object were also a factor in the success of college participants. These facts were as much apparent in the solutions of students in the modeling group as in those of students in the control group (Figure 8). Heller and Reif (1984) and Heller, Keith, and Anderson. (1992) have shown that such object and interaction descriptions and depictions are the most critical in solving physics problems. Students need to be taught explicitly and systematically to use schematic information shown in Figures 5 and 6 so that they can successfully: (a) describe the environment of an object, its object and interaction properties, its structure, and especially its behavior, and (b) depict them in appropriate force and motion diagrams. Such descriptions and depictions are not emphasized enough in traditional instruction, which leads students to ignore them in their solutions. Consequently, they fail to solve assigned problems correctly.

Two major stages of the modeling process are often ignored in traditional physics instruction. These are model validation and model deployment. In the tutorials, students were guided to ask themselves and answer related questions similar to the ones presented in the appendix. This has undoubtedly paid off in participants' reported success. Heller and Reif (1984) and Heller et al. (1992) have also argued for the importance of teaching students to "check and evaluate" their solutions.

Our interactive, dialectical approach could have been especially helpful for students to develop the modeling process, especially validation and deployment skills. After presenting a solution to an assigned problem, presenters were always asked during the tutorials to justify their solutions, and directed to engage in *Socratic* dialogues with their peers. We have shown elsewhere the importance of such an approach (Halloun & Hestenes, 1987), and so did Hake in laboratory contexts (1992). Teaming students up into groups should have also helped. Heller et al. (1992) have shown that cooperative grouping helps college physics students reach "better problem solutions...than...individuals working alone."

It was impossible in our experiment to assess each of the factors discussed above separately. However, the findings of other researchers discussed above suggest that every one of these factors had its contribution. Furthermore, it could have helped to have control groups that followed traditional tutorials during the same period in order to assess the effect of extracurricular instruction and the motivation of enrolled students (especially in college). However, previous research suggests that problem solving abilities of physics students, even the most motivated ones, do not improve significantly following extra traditional instruction that is not model-based (Halloun & Hestenes, 1987).

Acknowledgment

In the preparation of this article, I have benefited from discussions with many colleagues. Dale Baker, Mike Piburn, and Victor Pollak provided insightful comments on an earlier draft of this article. Larry Dukerich, Jane Jackson, and Gregg Swackhamer constantly lent their critical ears and incited me to refine and clarify my ideas. David Hestenes was all along behind this work and arranged the support that made it possible at this time. Last, but not least, the reported experiment could not have been possible without the collaboration of administrators, teachers and students at the participating institutions.

This work was supported in part by grants from the National Science Foundation in the USA and from Lebanese University in Lebanon.

References

AAAS. (1990). *Science for all Americans. Project 2061.* New York, NY: Oxford University Press.

AAAS. (1993). *Benchmarks for science literacy. Project 2061.* New York, NY: Oxford University Press.

American Mathematical Association of Two-Year Colleges. (1995). *Crossroads in mathematics. Standards for introductory college mathematics before calculus.* Memphis, TN: AMATYC.

Arons, A. (1981). Thinking, reasoning, and understanding. *The Physics Teacher, 19,* 166-172.

Bower, G.H., & Morrow, D.G. (1990). Mental models in narrative comprehension. *Science, 247,* 44-48.

Bronowski, J. (1953). *The common sense of science.* Cambridge, MA: Harvard University Press.

Casti, J. L. (1989). *Alternate realities. mathematical models of nature and man.* New York, NY: Wiley-Interscience.

Clement, J. (1989). Learning via model construction and criticism. In Glover, G., Ronning, R. & Reynolds, C. (Eds). *Handbook of creativity, assessment, theory and research.* New York, NY: Plenum.

Edwards, D., & Hamson M. (1989). *Guide to mathematical modelling.* Boca Raton, FL: CRC Press.

Eylon, B.S., & Reif. (1984). Effects of knowledge organization on task performance. *Cognition and Instruction, 1(1),* 5-44.

Gentner, D., & Stevens A. L., Eds. (1983). *Mental models.* Hillsdale, NJ: Lawrence Erlbaum.

Giere, R.N. (1988). *Explaining science: A cognitive approach.* Chicago, IL: University of Chicago Press.

Giere, R.N., Editor. (1992). *Cognitive models of science.* Minnesota Studies in the Philosophy of Science, Vol. XV. Minneapolis, MN: University of Minnesota Press.

Hafner, R.S. (1992). Teaching genetics: Recommendations and research. In Smith, M.U. & Simmons, P.E. (Eds.). *Teaching genetics: Recommendations and research. Proceedings of a National Conference*. Cambridge, MA.

Hafner, R., & Stewart, J. (1995). Revising explanatory models to accommodate anomalous genetic phenomena: Problem solving in the “context of discovery”. *Science Education, 79(2),* 111-146.

Hake, R. R. (1992). Socratic pedagogy in the introductory physics laboratory. *The Physics Teacher, 30,* 546-552.

Hake, R. (1994, August). *Survey of test data for introductory mechanics courses.* Paper presented at the Summer Meeting of the American Association of Physics Teachers, South Bend.

Halloun, I. (1986). Le réalisme naïf et l’apprentissage de la physique. *Recherches Pédagogiques, 17,* 23-47.

Halloun, I. (1995a, April). *Assessment of student beliefs about the world, physics and education.* Paper presented at the Thirty-second Conference of the Pacific Northwest Association for College Physics, Portland.

Halloun, I. (1995b, April). *Modeling theory in physics instruction.* Paper presented at the Annual Meeting of the National Association for Research in Science Teaching, San Francisco.

Halloun, I. (1996). *Schematic concepts for schematic models of the real world: The Newtonian concept of force.* Manuscript submitted for publication.

Halloun, I., & Hestenes, D. (1985). The initial knowledge state of college physics students. *American Journal of Physics, 53(11)*, 1043-1055.

Halloun, I., & Hestenes, D. (1987). Modeling instruction in mechanics. *American Journal of Physics*, *55(5)*, 455-462.

Hammer, D. (1989). Two approaches to learning physics. *The Physics Teacher*, *27*, 664-670.

Hammer, D. (1994). Epistemological beliefs in introductory physics. *Cognition and Instruction, 12(2)*, 151-183.

Heller, J.I., & Reif, F. (1984). Prescribing effective human problem solving processes: Problem description in physics. *Cognition and Instruction, 1(2)*, 177-216.

Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. *American Journal of Physics, 60(7)*, 627-636.

Hesse, M.B. (1970). *Models and analogies in science.* South Bend, IN: University of Notre Dame Press.

Hestenes, D. (1987). Toward a modeling theory of physics instruction. *American Journal of Physics*, *55(5)*, 440-454.

Hestenes, D. (1992). Modeling games in the Newtonian world. *American Journal of Physics*, *60(8)*, 732-748.

Hestenes, D. (1995). Modeling software for learning and doing physics. In Bernardini, C., Tarsitani, C., & Vincentini, M. (Eds.). *Thinking physics for teaching.* 25-66. New York: Plenum.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher*, *30*, 141-158.

Johnson-Laird, P.N. (1983). *Mental models*. Cambridge, MA: Cambridge University Press.

Leatherdale, W.H. (1974). *The role of analogy, model and metaphor in science.* New York, NY: American Elsevier Pub.

Mac Lane, S. (1988). Mathematical models: A sketch for the philosophy of mathematics. *The American Mathematical Monthly*, *95*, 462-471.

McDermott, L. (1993). How we teach and how students learn – A mismatch? *American Journal of Physics, 61(4)*, 295-298.

Mestre, J.P., Dufresne, R.J., Gerace, W.J., Hardiman, P.T., & Touger, J.S. (1993). Promoting skilled problem-solving behavior among beginning physics students. *Journal of Research in Science Teaching, 30(3)*, 303-317.

National Resesarch Council. (1996). *National science education standards*. Washington, DC: National Academy Press.

Nersessian, N.J. (1995). Should physicists preach what they practice? Constructive modeling in doing and learning physics. *Science & Education, 4(3)*, 203-226.

Novak, J.D. Editor. (1987). *Proceedings of the Second International Seminar. Misconceptions and Educational Strategies in Science and Mathematics*. New York, NY: Cornell University.

Novak, J.D. Editor. (1994). *Proceedings of the Third International Seminar. Misconceptions and Educational Strategies in Science and Mathematics*. New York, NY: Cornell University.

Pollak, V.L. (1994). Science education II: Scientific literacy and the Karplus taxonomy. *Journal of Science Education and Technology, 3(2),* 89-97.

Raghavan K., & Glaser, R. (1995). Model-based analysis and reasoning in science: The MARS Curriculum. *Science Education, 79(1),* 37-61.

Redish, E. (1994a, January). *The Distribution of Student attitudes towards doing physics.* Paper presented at the Winter Meeting of the American Association of Physics Teachers, San Diego.

Redish, E. (1994b). Implications of cognitive studies for teaching physics. *American Journal of Physics, 62(9),* 796-803.

Reif, F., & Allen S. (1992). Cognition for interpreting scientific concepts: A study of acceleration. *Cognition and Instruction, 9(1),* 1-44.

Reif, F., & Larkin, J.H. (1991). Cognition in Scientific and Everyday Domains: Comparison and Learning Implications. *Journal of Research in Science Teaching, 28(9)*, 733-760.

Smith, M.U. (1992). Implementing a conceptual basis for teaching and learning classical genetics. In: *Teaching genetics: Recommendations and research. Proceedings of a National Conference.* 69-79. Cambridge, MA.

Steen, L.A. Editor (1990). *On the shoulders of giants. New approaches to numeracy.* Washington, DC: National Academy Press.

Stewart, J., & Hafner, R. (1991). Extending the conception of "problem" in problem-solving research. *Science Education, 75(1)*, 105-120.

Stewart, J., Hafner, R., Johnson, S., & Finkel, E. (1992). Science as model building: Computers and high-school genetics. *Educational Psychologist, 27(3),* 317-336.

Strnad, J. (1986). On some new trends in physics teaching. *European Journal of Physics, 7,* 11-16.

Swann, W.F.G. (1950). The teaching of physics. *American Journal of Physics, 19(2),* 182-187.

Swetz, F., & Hartzler, J.S., Eds. (1991). *Mathematical modeling in the secondary school curriculum.* Reston, VA: National Council of Teachers of Mathematics.

Tobias, S. (1990). *They're not dumb, they're different. Stalking the second tier*. Tucson, AZ: Research Corporation.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics, 63(7),* .

White, B.Y. (1993). Intermediate causal models: A missing link for successful science education? In Glaser, R. (Ed.). *Advances in Instructional Psychology, 4*, 177-251.

White, B.Y., & Frederiksen, J.R. (1990). Causal model progressions as a foundation for intelligent learning environments. *Artificial Intelligence, 42*, 99-157.

Appendix

Modeling for solving mechanics problems

Model Selection*

- What physical *systems* are described in the problem? What objects does each system consist of? What objects outside each system interact with those inside?
- What *type of motion* does each system undergo (or could undergo, if it were at rest)? Translation? Rotation? Of what kind?
- In what *reference system*?
- What *model* is most appropriate for modeling each physical system?

* In solving a paradigm problem, students should be encouraged to select a model tentatively at first, based on the description of the situation in the problem's statement, without reading the questions. This should help students learn: (a) that solving a physics problem is *not a formula selection process*, and (b) that the mathematical solution of a physics problem follows from the correct choice of respective models.

Model Construction

- What *coordinate system* best depicts the chosen reference system?
- What *parameters* are required for each object (mass, moment of inertia)?
- What *kinematical concepts* are required to describe the motion of each object? What axioms/definitions are required for each concept?
  Depict those concepts in an appropriate *motion map*.
- What *kinematical laws* describe the translation/rotation of each object?
  Depict them mathematically: Write the corresponding mathematical equations, complete the motion map accordingly, and/or draw appropriate graphs, if necessary.
- What *agents* act on each object? What *force* is exerted by each agent?
  Depict those forces in an appropriate *force diagram.*
- Are there any kinematical/dynamical constraints? Initial/final conditions?
  If any, specify and express them mathematically.
- Are any *interaction laws* necessary to quantify the forces?
- What *causal laws* explain the state of each object (rest, translation, rotation)? Conservation laws may be more appropriate for time-independent situations, Newton's laws and/or Euler's laws for time-dependent ones.
  Express them mathematically.

Now, students may be directed to read the first question in the problem, and repeat the following steps for every question.

Model Validation

Correspondence (or reference) assessment:

- Can the model(s) thus constructed adequately represent the object(s) in the problem?
- Is the question asked in the problem within the domain(s) of the chosen model(s)?

Completeness assessment:

- Have all primary kinematical/dynamical properties of each object been represented in the corresponding model?
- Are there any secondary properties represented in a model that should have been ignored?
- Is the mathematical model thus constructed sufficient to answer the question?

Internal consistency assessment:

- How well does each mathematical representation match what it represents?
- Do related kinematical concepts, and their depictions, match each other?
  Example: Check whether an acceleration correctly describes the change in the corresponding velocity.
- Do kinematical concepts correspond to dynamical concepts, and do their depictions match each other?
  Example: Check whether acceleration and force vectors match each other.
- Are various kinematical/dynamical laws (and their depictions) coherent?
- Are constraint conditions verified?
- Are dimensions/units verified in all relationships?

External consistency assessment:

- Is the model thus constructed consistent with those previously constructed for studying similar situations?

Sensitivity assessment:

- Is the model sensitive to differences between different objects in the problem?

Fidelity assessment:

- Could neglected features of any object significantly affect the outcomes?

Outcomes assessment (following model analysis):

- How does the mathematical solution answer the questions in the problem?
- How well do outcomes correspond to empirical evidence?
- Are outcomes reproducible using a different approach?

Model deployment

- What elements of the model were *critical* for solving the problem?
- In what respects is this problem *similar to / different from* other problems that you thought at first might be similar / different? How about its solution?
- What aspects in the problem and its solution *reinforce* some of your knowledge?
- What aspects in the problem and its solution *complement* some of your knowledge?
- What aspects in the problem and its solution *contradict* some of your knowledge?
- What aspects in the problem and its solution are *novel* to you?
- How can the problem and its solution be deployed into *other situations*?