Schematic Concepts for Schematic Models of the Real World

The Newtonian Concept of Force

Ibrahim Halloun
Department of Physics & Astronomy, Arizona State University
Box 871504, Tempe AZ 85287-1504
On leave from Lebanese University

Abstract

High school and college students often carry out of traditional physics courses loose bundles of vague and undifferentiated concepts about physical objects and their properties. Within the framework of Schematic Modeling, a scientific concept can be defined explicitly with five schematic dimensions: domain, organization, quantification, expression and employment. Based on the level of commensurability between scientific concepts and individual students' own concepts, students' cognitive evolution into the scientific realm can take different directions ranging from reinforcing existing concepts to constructing novel ones on completely new foundations. Such evolution is promoted in a student-centered, model-based instruction. The Newtonian concept of force is discussed for illustration, along with the results of tutoring two groups of Lebanese students to develop this concept in a schematic modeling approach.

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Scientific literacy is nowadays a necessity for the general public; yet, a majority of students at all levels shy away from taking science courses, and those who do take science often pass their courses without meaningful understanding of the subject matter (Barrington & Hendricks, 1988; Edmondson & Novak, 1993; Halloun & Hestenes, 1996; Meichtry, 1993; NCES, 1994; Songer & Linn, 1991). The state of physics education is especially alarming in this respect. Teachers at all levels still echo the same complaint that Swann expressed about half a century ago, that a physics student "passes his [or her] tests frequently alas, with very little comprehension of what he [or she] has been doing" (Swann, 1950). Calls for major reform in physics education are thus being raised increasingly, and model-based instruction is being advocated by many educators to this end.

High school and college students often come out of their physics courses the way they came in, unable to: (a) realize how a physics concept or principle relate to the real world, (b) differentiate among different concepts, (c) relate individual concepts to each other, (d) develop appropriate procedures for applying a concept or a principle to real world situations, and (e) express themselves correctly when trying to engage in scientific discourse (Cobern, 1993; Halloun, 1996a; Halloun & Hestenes, 1985a, 1996; Hammer, 1994; Mazur, 1996; McDermott, 1993; Reif & Allen, 1992; Reif & Larkin, 1991).

Teachers more than students should hold the blame for students' deficiencies. Teachers often have wrong expectations about their students especially when they teach by lecture and demonstration. In the traditional lecturing approach, physics courses are wrongly treated as *transferable commodities* that students can readily consume on their own. Moreover, concepts and principles of physics are often presented *episodically*. One concept or principle is presented after another without necessarily showing how they all relate to one another in coherent structures, and how they can be systematically used for describing, explaining, predicting, controlling, and/or designing real world systems and phenomena.

Alerted to the gravity of the situation, educators and concerned groups started recently to look for alternatives to the traditional, episodic lecturing approach, and stress the need to bridge the epistemology of science with cognition for meaningful science education reform (Duschl & Hamilton, 1990). A common call has been to *restructure* the materials presented in science courses in a manner that reflects the nature of scientific knowledge, and to *engage students actively* in learning processes that build on the students' initial knowledge state (AAAS, 1990 & 1993; NCEE, 1983; NRC, 1996; NSTA, 1993 & 1995).

Model-based instruction has been presented as a viable alternative to traditional instruction in this direction. Accordingly, scientific concepts and principles are developed and coherently related to one another within the context of conceptual models. Modeling advocates come from all scientific disciplines; from biological sciences (Hafner, 1992; Hafner & Stewart; 1995; Smith, 1992; Stewart & Hafner, 1991; Stewart, Hafner, Johnson, & Finkel, 1992), to physical sciences (Clement, 1989; Halloun & Hestenes, 1987; Hestenes, 1987, 1992 & 1995; Nersessian, 1995; Pollak, 1994; Raghavan & Glaser, 1995; Redish, 1994; Wells, Hestenes & Swackhamer, 1995; White, 1993; White & Frederiksen, 1990), as well as mathematics (AMATYC, 1995; Casti, 1989; Edwards & Hamson, 1989; Mac Lane, 1988; Steen, 1990; Swetz & Hartzler, 1991). Without necessarily agreeing on the details, many of those educators have shown that students reach significantly better understanding of their courses under model-based instruction than under traditional lecturing.

Schematic Modeling is a model-based, student-centered pedagogical framework. Models and modeling processes are at the heart of the epistemology of schematic modeling. In the classroom, students are constantly guided through reflective and interactive modeling activities whereby their initial knowledge is considered a cognitive stepping stone on their way to the scientific world. The epistemological tenets of this new pedagogical framework have been presented elsewhere, along with detailed characteristics of scientific models and modeling processes in science education (Halloun, 1996b).

In this paper, the epistemology of scientific *concepts* is presented from a schematic modeling perspective, and students' initial cognitive states are discussed based on the level of *commensurability* between the students' own concepts and their scientific counterparts. General instructional guidelines are also proposed to help students avoid the accumulation of fragmented concepts and evolve into the coherent and systematic world of science. The concept of force is used for illustration within the context of basic Newtonian models of mechanics. The educational potentials of guiding students to develop this concept in a schematic modeling approach are last discussed while overviewing results of tutorials conducted in Lebanon following this approach. But first, and in order to put things into perspective, let us briefly review the major tenets of schematic modeling and the main characteristics of schematic models and modeling.

Schematic Modeling: An Overview

Various forms of model-based instruction have been reported in the literature. However, and as we have argued elsewhere, little effort has been devoted to the formulation of a comprehensive pedagogical framework for such type of instruction, and no consensus has yet emerged as to what models and modeling are all about (Halloun, 1996b). Such lack of consensus can actually be traced in some respects to modeling advocates within the philosophy of science and cognition communities whose works lend to *schematic modeling* its major tenets.

Before we review those tenets, let us first point out a major premise of our pedagogical framework. In formal education, as in any intellectual activity, we distinguish between two levels of thought inherent in the so-called schemata and constructs, two distinct terms that are often used loosely and interchangeably.

Schemata hereby refer to *mental* structures that are mapped directly onto the physical wirings of the brain. They are *tacit* and cannot be explored directly. Schemata are always *subjective*. They depend on the idiosyncrasies of a person, the unique nature of her/his neural networking and social experience.

Constructs, on the other hand, hereby refer to *conceptual* structures that one can communicate with others *explicitly* in one form or another (verbal, pictorial, mathematical, etc.). Although generated by individuals' schemata, constructs can be *objective*, especially when they reflect the shared meanings and common understandings of a professional community. The documented scientific knowledge consists of scientists' constructs (concepts, laws, models, theories) and not of their schemata.

So far, schemata have mostly been the object of study in *cognition* and *neuroscience*, and constructs, the object of study in *epistemology*. Schemata are more fundamental than constructs, for they are at the basis of the latter. However, because of their tacit nature, and until cognition and neuroscience can show us differently, schemata can only be explored indirectly through constructs. Consequently, the ultimate aim of science education to empower students with schemata that are compatible with scientists' schemata can only be achieved indirectly through the negotiation of constructs, and with the hope that a measurable *conceptual evolution* is actually the result of the desired *mental evolution*.

In this article, we thus take the position of comparing student and scientist schemata in terms of "compatibility", and their respective constructs in terms of "commensurability" in order to stress at least two major points. The first is that constructs but not schemata are directly "measurable". The second is that student constructs and their evolution cannot be evaluated in the abstract but by comparison to desired standards. These standards are schematic, scientific constructs.

The tenets of schematic modeling, the characteristics of scientific models and modeling processes have been presented in details elsewhere (Halloun, 1996b). An overview of these topics is presented here in order to allow the reader to follow our discussion of schematic concepts, the main theme of this article.

Major tenets

The following are major tenets of *schematic modeling* (Halloun, 1996b):

1. "We build mental models that represent significant aspects of our physical and social world, and we manipulate elements of those models when we think, plan, and try to explain events of that world" (Bower & Morrow, 1990).
2. "Our view of the world is causally dependent both on the way the world is and on the way we are. There is an obvious but important corollary: all our knowledge of the world depends on our ability to construct models of it" (Johnson-Laird, 1983).
3. One's conceptual evolution (and hopefully mental evolution) can be efficiently and meaningfully achieved in a *middle-out approach* at the core of which are conceptual models. From conceptual models, one can go up the epistemological ladder to build a general theoretical framework, or down to develop individual concepts.
4. *Scientific* models are the objective and coherent conceptual models par excellence (Bronowsky, 1953; Casti, 1989; Giere, 1988, 1992; Hesse, 1970; Hestenes, 1995; Leatherdale, 1974; Nersessian, 1995; Redish, 1994).

   Scientific models are *schematic* in the sense that, like other scientific constructs, they are: (a) *reduced* to a limited number of *primary* features, i.e. features that are relevant to the object of study and that are almost independent of the idiosyncrasies of individual scientists, and (b) developed and applied following *generic* modeling *schemes,* i.e., systematic plans of interaction with the real world that enable one to construct and employ not only some models of interest but all sorts of new constructs.
5. A *few basic* conceptual models are at the foundations of every scientific theory. A mastery of these models is indispensable for meaningful understanding of individual concepts and principles in the theory, and thus for evolving into the scientific world (Giere, 1994; Wells et al., 1995).
6. A student's evolution into the scientific world requires the adoption of a scientific epistemology that is often different in many respects from the student's own epistemology (Gentner & Stevens, 1983; Hestenes, 1992; Mortimer, 1995; Nersessian, 1995).

   This evolution cannot take place unless students are motivated to evaluate their own constructs while they are engaged interactively in modeling physical world systems and phenomena (Hake, 1992 & 1996; Halloun & Hestenes, 1987; Heller, Keith & Anderson, 1992; Wells et al., 1995).

Two major implications for science education that are the object of this paper follow from the tenets above:

1. Science teachers should account explicitly for students' initial knowledge state, and refrain themselves from the mere transfer of scientific information. They should guide their students to resolve any incommensurability between their own constructs and their scientific counterparts.
2. Scientific concepts should be developed systematically, and not episodically, within the context of schematic, basic models.

Schematic models

Whatever pedagogical philosophy an educator subscribes to, the target knowledge that students should develop needs to be specified ahead of time. Otherwise, students would be left to wander in futile activities. The transmission of scientific knowledge as inscribed episodically in science textbooks has been the main goal of traditional science instruction. In contrast, within the framework of schematic modeling students are guided to develop generic scientific reasoning skills and a coherent view *of and about* scientific knowledge. Such coherence is promoted in a middle-out approach at the core of which are scientific models. From models, students are guided up the epistemological ladder to build scientific theory, and down to develop individual concepts.

Models are considered as unifying themes in recently published science education standards (AAAS, 1990, 1993; NRC, 1996). Concepts are to conceptual models as a quark or an electron are to an atom. No atom can exist without these elementary particles. However, comprehensive and meaningful understanding of these particles can only be gained within the context of atomic structure and phenomena. Similarly, and according to schematic modeling, meaningful understanding of individual scientific concepts is best achieved within the context of schematic, and especially basic, models.

According to schematic modeling, a comprehensive presentation of a scientific model (hereafter referred to as "model") can be brought about in four dimensions: domain, composition, structure, and organization. Composition and structure "define" a model. Domain and structure situate it in the theory to which it belongs. The four dimensions are briefly presented below and illustrated in Figure 1 to set the stage for schematic concepts, the main topic of this article. Details can be found elsewhere (Halloun, 1996b).

Insert Figure 1 about here

The *domain* of a model consists of physical systems and phenomena that can be described and/or explained, in some respects, and to a certain degree, using the model.

The *composition* of a model consists of conceptual objects and agents, and respective properties or descriptors.

The *structure* of a model consists of *relationships* among the descriptors of different entities. Depending on the type of structure included in the model, a model can be descriptive and/or explanatory. Kinematical models of Newtonian mechanics are descriptive, dynamical models are explanatory.

The *organization* of a model refers to its relationship to other models in a given scientific theory. Every theory provides appropriate *classification schemes* for grouping various models into *families* of models, the most fundamental of which are the families of basic models.

A *basic model* is one with simple composition (often consisting of one object) and simple structure (often limited to describing and/or explaining one elementary phenomenon). Figure 2 shows two families of basic models in classical mechanics: the family of basic *particle models,* and the family of basic *rigid body models.*

Insert Figure 2 about here

The four schematic dimensions presented above constitute for a model what is often referred to in the literature as *declarative knowledge*. These dimensions are complemented by some *procedural knowledge*. Procedural knowledge consists of the rules that guide the modeling processes outlined next, and associated with it is a repertoire of familiar situations in which a person had the chance to successfully apply the model. Reif (1987) calls this sort of repertoire "compiled knowledge", and recommends to resort to it mostly "when one encounters a familiar situation". However, we hold that one can also resort to compiled knowledge in unfamiliar situations, especially when they can be dealt with using *bridging analogies* in a manner described by Clement (1993).

Modeling

Modeling a physical situation is a systematic process that follows a five-stage scheme outlined below. Details about this modeling scheme can be found elsewhere (Halloun, 1995, 1996):

1. ***Model selection.*** Individual systems are first conveniently defined in the situation. Then an appropriate model is built to represent each system within the framework of an appropriate scientific theory. The model can be entirely new or chosen from the person's repertoire of models.
2. ***Model construction.*** The composition and structure of each model are specified and expressed mathematically in the form of a preliminary mathematical model.
3. ***Model validation.*** Each model is assessed to determine whether it fulfills as constructed some specific needs in the situation, or needs to be refined or even replaced to this end.
4. ***Model analysis.*** Once a model is validated, analysis can proceed to fulfill the purpose for which it is being constructed. In solving textbook problems, model analysis consists primarily of processing a mathematical model, getting answers to the questions asked in the problems, and interpreting and justifying the answers.
5. ***Model deployment.*** Once a model is validated and analyzed in a particular situation, one needs to extrapolate the new experience. Such extrapolation includes *fine-tuning* any of the schematic dimensions of the model and involved concepts, setting conditions for *transferring* the model to new situations (i.e., expanding the compiled knowledge), and integrating the refined model in one's own knowledge base.

Schematic Concepts

Now that we have reviewed relevant underpinnings of schematic modeling, let us turn to individual concepts and discuss respective epistemological and cognitive issues within the context of the newly proposed framework. First, the epistemological issues.

Three types of concepts can be distinguished in the epistemology of science, and especially physics: object concepts, property concepts, and operational concepts. *Object concepts* like the concept of particle in mechanics refer to physical objects in the real world. *Property concepts* like the concepts of speed or force refer to physical properties that are particular to a given physical object (speed) or that characterize its interaction with other physical objects (force). *Operational concepts* like vector addition are logico-mathematical concepts that are used to process object and property concepts. In this article we concentrate on property concepts referred to hereafter simply as "concepts" or *descriptors*.

Two types of descriptors can be distinguished: object or individual descriptors and interaction descriptors (Reif & Heller, 1982). An *object descriptor* refers to a characteristic feature of a given physical object. This feature can be *intrinsic* (e.g., mass), or *state-dependent* (e.g., speed). An *interaction descriptor* (e.g., force) refers to a feature characterizing the interaction between at least two physical objects. In the following we concentrate on interaction descriptors, and mainly the concept of force in Newtonian theory. The concept of force is chosen because it is the most fundamental interaction concept in introductory physics courses, and about the most difficult for students to understand (Halloun & Hestenes, 1985a; Hestenes, Wells & Swackhamer, 1992).

Schematic concepts: an epistemological perspective

In traditional physics textbooks, different concepts are presented episodically, often in different and unsystematic ways, making it difficult for students to realize what is it that they need to learn about any given concept. Some physics educators have recently attempted to remedy this deficit by proposing explicit epistemological frameworks for systematic and coherent presentation of *scientific* concepts. Among those educators are Reif and his colleagues who argue that "good scientific concept interpretation requires that the name of the concept be accompanied by several main types of interpretation knowledge, each with associated ancillary knowledge" (Reif & Allen, 1992). Other educators have instead concentrated on developing frameworks for characterizing *students'* knowledge. Among the latter are Minstrell and diSessa who suggest that students' mental evolution is determined by the evolution of some elementary cognitive units that control behavior. Minstrell (1991) calls these units "facets", and defines a facet as a "convenient unit of thought, a piece of knowledge or a strategy seemingly used by the student in addressing a particular situation". DiSessa (1993), on the other hand, calls these units *phenomenological primitives*, or p-prims, and defines them as "primitive elements of cognitive mechanism...that act largely by being recognized in a physical system or in the system's behavior or hypothesized behavior...They are ready schemata in terms of which one sees and explains the world". The works in both camps are insightful and deserve the careful attention of science educators.

In this section, we propose a comprehensive way for defining scientific descriptors, and illustrate with the Newtonian concept of force. In the next section, we establish different levels of commensurability between student and scientist concepts, based on which we propose general instructional guidelines.

According to the third and fifth tenets of schematic modeling presented above, a concept can be meaningfully and efficiently constructed within the framework of *basic models*. In the Newtonian world, this implies that an interaction concept like the concept of force needs to be constructed within the framework of particle models (Figure 2).

Following the fourth tenet and the schematic dimensions of a model, we propose that a scientific concept be defined comprehensively in five schematic dimensions discussed below: domain, organization, quantification, expression, and employment.

The proposed schematic dimensions are meant to serve as a comprehensive template for planning and evaluating instruction, and putting more structure and coherence in the presentation of various concepts. Information contained in the five dimensions needs to be presented entirely to the students, but not necessarily broken along the lines of those dimensions. In fact, the dimensions should not even be presented as such to students, at least not freshmen. As instruction progresses, teachers may encourage students to develop some kind of a prescription consisting of a list of questions that one needs to answer in order to develop a schematic concept comprehensively.

Domain

Research in physics education has shown that students' constructs about the physical world are often incommensurable with scientific constructs in many respects. One critical deficit reported in students' knowledge is in the mapping between their concepts and the physical world. Student concepts are often situation-specific and concentrate on irrelevant features of physical objects. Students often use the wrong descriptor to refer to a given physical property or use different descriptors interchangeably (Chi, Feltovich & Glaser, 1981; Halloun & Hestenes, 1985-a & b, 1996; Hammer, 1994; Novak, 1987, 1994; Reif, 1987; Reif & Allen, 1992). These problems can be attributed to the failure of traditional instruction to help students explicitly delimit the domain of applicability of individual constructs.

The domain of a scientific descriptor consists of a set of *referents* (physical entities of the real world) which share a specific feature that is represented in some respects and to a certain degree by the descriptor in question. The conditions and limitations of applicability of the descriptor to its referents can be formulated in a set of *correspondence rules.*

The concept of *force* characterizes the *interaction* between two distinct physical entities referred to as *object* and *agent* in models of Newtonian mechanics. Two types of interaction are commonly distinguished in introductory classical mechanics courses: (a) long-range interaction or interaction at a distance, and (b) contact interaction. Figure 3 shows some common agents and respective forces, along with vectorial depictions (discussed below) of such forces when exerted on particle-like objects. Figure 4 shows some correspondence rules associated with the Newtonian concept of force.

Insert Figures 3 and 4 about here

Organization

Due to the episodic nature of traditional physics instruction, student concepts are weakly structured and fragmented (Halloun & Hestenes, 1985-a & b, 1996; Hammer, 1994; McDermott, 1993; Novak, 1987, 1994; Reif, 1987; Reif & Allen, 1992). This deficit can be remedied by making explicit the role of a given concept in the construction of basic models.

An isolated concept is practically meaningless and useless. A concept is always related to other concepts in scientific theory through axioms, definitions and/or laws, the network of which make up the organization of the concept. The *structure* of basic models provide the essential context for laying out such organization (Figure 1).

Depending on their complexity, two types of property concepts can be distinguished: prime and derived. *Prime* concepts are those that cannot be derived from other concepts. *Derived* concepts are those that are commonly "defined" *explicitly* in terms of prime concepts and/or other derived concepts. The concept of force is a prime concept, whereas the concept of work is a derived concept.

A prime concept is commonly "defined" *axiomatically,* i.e., implicitly through a given set of axioms or laws. Such is the case with the Newtonian concept of force which is commonly defined axiomatically through the entire set of Newton's laws of dynamics. For this reason, these laws are sometimes referred to as axioms of force.

Newton's 2nd law is sometimes wrongly presented as *the definition* of the concept of force. This law is just that: a law, not a definition. A "definition", in the traditional sense, relates concepts of the same nature, kinematical *or* dynamical in Newtonian theory (e.g., the definition of velocity in terms of position, or of work in terms of force). Newton's second law is a *causal law* that *explains* the change of state of an object (as defined by its momentum or velocity).

Along with Newton's laws of dynamics, *interaction laws*, such as Newton's law of universal gravitation, Coulomb's law of electrostatic interaction or Hooke's law, set out the organization of the force concept within the framework of Newtonian models of mechanics, and especially the families of basic models (Figures 1 and 2).

Quantification

Physics students are often unable to measure a given descriptor appropriately. Their problems include: (a) mixing between scalar and vectorial quantities such as speed and velocity, (b) failure to assign the right unit to a descriptor or to convert between different unit systems, (c) inability to make appropriate estimations or to realize when the numerical value of a descriptor is out of its allowed range, (d) conducting inappropriate mathematical operations such as adding forces applied on two different objects in applying Newton's second law (Arons, 1984, 1993; Gunstone, 1991; Reif & Larkin, 1991; Van Heuvelen, 1991; Viennot, 1985). These problems could have been avoided had traditional physics instruction made it explicit to students how they should go about quantifying physical descriptors.

A descriptor cannot belong to physics unless it is quantifiable. Quantification of physics descriptors is necessary to better understand them and to ensure their objectivity. A century ago, Lord Kelvin (1891) argued:

> "when you can measure what you are speaking about, and express it in *numbers*, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thought, advanced to the stage of science".

Quantification of a concept is done according to laws and following rules that are set by the theory to which the concept belongs. *Quantification laws* set the quantitative nature of a concept, the operations that can be undertaken with it, and the assumptions underlying its measurement. *Quantification rules* specify how to practically measure the concept and determine the respective limits of approximation and precision. Figure 5 shows some quantification laws and rules corresponding to the concept of force within the framework of Newtonian theory.

Insert Figure 5 about here

Expression

Objectivity of scientific concepts extends to the way they are communicated among scientists. Physicists express each concept of physics in specific ways that distinguish it from other concepts. The words "force", "acceleration", and "energy", for example, are for physicists the names of different descriptors that refer to different physical properties in the real world. However, students often use anyone of the three words to refer interchangeably to either concept, or mix them up in a way that reflects an actual indiscrimination among the various concepts.

Expression *means* or forms of a given concept of physics include its identification, symbols, labels, pictorial depictions, and mathematical representations, along with the corresponding *semantics* for interpreting the various forms of expression. Figure 6 shows expression means and semantics for the Newtonian concept of force.

Insert Figure 6 about here

Employment

Students often memorize information about physics descriptors by rote, including the ways a concept is used in exemplary situations. These ways are regarded as situation-specific recipes that can be recalled and reproduced automatically when confronted later with the same or identical situations (Arons, 1984; Chi et al., 1981; Halloun & Hestenes, 1996; Larkin, McDermott, Simon & Simon, 1980; Reif, 1987; Reif & Larkin, 1991; Strnad, 1986; Van Heuvelen, 1991).

A scientific concept can be used to characterize its referents, as well as to develop new constructs. New constructs extend from derived concepts to conceptual models. The employment of any concept is guided by appropriate rules that stem from the above four schematic dimensions and that are set by the theory to which it belongs.

The most critical part of employing a concept is to decide whether or not it is suitable for an appropriate situation. This choice is guided by the correspondence rules of various concepts, and facilitated by the repertoire of familiar situations in which a person has successfully employed each concept (compiled knowledge). Once a concept is determined suitable, one basically needs only to *reproduce* part of, or the entire schematic dimensions of this concept, provided that one has already learned such dimensions. Such reproduction is often done within the context of a model.

The employment of the concept of force is governed by the previous four schematic dimensions as outlined in Figures 3 through 6, and the models in which it is being used. In the case of classical mechanics, Figure 3 provides some guidelines for identifying agents interacting with a specific object and depicting the corresponding forces. Figure 7 shows some other guidelines that need to be followed when employing the Newtonian concept of force with basic particle models (Figure 2).

Insert Figure 7 about here

Student and scientist concepts: a cognitive perspective

The epistemology of schematic modeling is complemented by cognitive guidelines for instruction in an attempt to make it a comprehensive pedagogical framework. These guidelines stem from current developments in science education without necessarily subscribing to any of the predominant schools.

A middle-out approach of mental evolution is promoted in a model-based, student-centered instruction. According to tenets 1, 2, and 6 of schematic modeling, meaningful understanding of science can be reached only when students are afforded to reflect back on their own schemata and make them more compatible with scientific schemata. However, and as discussed earlier, this mental evolution can be facilitated only indirectly through the negotiation of constructs such as concepts and models.

For the negotiation of constructs to be conducted meaningfully and successfully, science instructors need to be versed in the epistemological foundations of scientific theory and cognitive foundations of the learning process. Instructors need to be especially aware of the initial knowledge state of their students, and of the processes that can facilitate the students' evolution into the scientific realm. Components of student initial state that are especially relevant include students' constructs about the subject matter, as well as their learning styles and general views about knowing and learning science.

We have presented elsewhere detailed descriptions of student initial knowledge state about the concept of force (Halloun, 1986; Halloun & Hestenes, 1985-a & b; Hestenes et al., 1992), as well as of their learning styles and views about science (Halloun, 1996a; Halloun & Hestenes, 1996). In the following, and using the concept of force for illustration, we concentrate on the various cognitive states that arise from the nature of student constructs about the subject matter, and the way instructors could deal with these states. Other cognitive issues related to student learning styles and views about science are beyond the scope of this paper.

Conceptual commensurability and instruction

Based on the degree of commensurability with scientific constructs, student initial constructs can be of three types. Some initial constructs may be *commensurable* with the desired scientific constructs. Others may be *incommensurable* in various respects not only with scientific constructs but also with one another. Finally, some of the desired constructs may have no counterparts in a student's knowledge base, and thus are *novel* to the student in the strict sense of the word. As outlined in Figure 8, the three types of constructs need to be treated differently in science instruction.

Insert Figure 8 about here

Figure 8 outlines a *dialectical* approach of instruction whereby individual students are guided to first become aware of the state of their own knowledge and then evolve through the evaluation of their constructs and the negotiation of their meanings and validity with external evidence and with their peers. As indicated in this figure, when the desired scientific construct does not have a counterpart in the student knowledge base (a situation that does not arise in the case of force), the teacher needs to do a lot of scaffolding by presenting many physical situations from which a common pattern can be inferred, resulting in the construction of the new construct. Subsequently, the student needs to reinforce the newly built construct in a manner described below.

Predominant schools that have had their merits in science education, like *conceptual change* and *constructivism*, have sometimes led educators to believe that novel constructs can be treated at the same cognitive footing as constructs that have some counterparts in student knowledge base. Dreyfus, Jungwirth and Eliovitch (1990) have demonstrated how far from the truth this is. The authors have shown for instance that when "input knowledge...has no counterpart in the experience of the student", guiding students through a process of conceptual change can be a futile endeavor, even when students get first dissatisfied with existing conceptions, and the new conceptions are intelligible, plausible and fruitful, as recommended by Posner, Strike, Hewson and Gertzog (1982).

When a student has a construct that is commensurable with the targeted scientific construct, the student needs to be guided to *reinforce* and *expand* the existing construct (Figure 8). For example, in the case of the concept of force, a student may already know what this descriptor refers to, but his/her force catalog may not be as comprehensive as the one shown in Figure 3. This student needs then to be engaged in activities that would help him/her extend his/her force catalog and the associated compiled knowledge. These activities may be of the type recommended above for novel constructs. But then reinforcement should follow with situations that are contextually rich and that allow the student to delineate salient from trivial features of every new agent. The compiled knowledge should include not only situations where the concept of force applies within the framework of Newtonian theory, but also situations where it does not apply (e.g., non-inertial reference frames). The student can then clearly delimit the domain of the concept and integrate it coherently in basic Newtonian models.

In the case of incommensurability between a student construct and the desired scientific one, two situations may arise that need to be treated differently. In one instance, the student construct may have some flaws without being entirely wrong. We then say that the student construct is underlined at the mental level by a *paraconception.* This is for instance the case when a student believes, based on everyday observations of objects falling in air, that objects of the same mass and different shapes are subject to gravitational interactions of different magnitudes. The student needs then to realize that, say, a flat sheet of paper falls "slower" than an identical sheet crumpled into a ball not because of different gravitational pulls but because of different drag forces exerted by the air. Hashweh (1986), and Smith, diSessa and Roschelle (1993) describe processes that we believe are appropriate to resolve the incommensurability in such instances (and only in such instances as we shall argue below). Students need to be engaged here not in a process of *conceptual change* to replace their constructs, but in a process of *refinement*. As Smith et al. (1993) argue, students need to be "encouraged to consider the limits of their conceptions" without denying the validity of these conceptions, and to get engaged in activities that allow them to use what they "already know in more general and powerful ways" and learn "where and why pieces of knowledge that are conceptually correct may only work in more restricted contexts".

In some instances, the student construct may actually be entirely *wrong* from a scientific perspective. We then say that the student construct is mentally underlined by a *misconception* (notwithstanding the objections that some may have to this term), and a true *conceptual change* is then needed for the student to evolve into the scientific realm. This is the case when a student believes for example that terrestrial objects fall only because air pushes them down (Halloun & Hestenes, 1985a). Students holding this belief are *wrong* in at least two counts. First, they actually do attribute the gravitational pull to the air and not the earth (wrong agent). Second, they consider air to be conducive of motion instead of resistive (wrong direction of air drag). Unlike students with paraconceptions, students encumbered with misconceptions need to *replace* their constructs with scientific ones. Such a replacement can take place in processes similar to the ones described by Minstrell (1989, 1991) or Dykstra, Boyle and Monarch (1992).

In both instances of incommensurability, the situations that the student needs to explore should allow a check for internal coherence with other constructs the student possess, as well as for predictive validity against physical phenomena. Minstrell (1982, 1989) report on typical student-teacher discourse that facilitate such checks of internal coherence and predictive validity. Furthermore, the resolution of incommensurability needs to be followed up by a reinforcement process similar to the one described above for commensurable concepts (Figure 8).

Classroom activities

The conceptual evolution discussed above can be meaningfully achieved when students are motivated to negotiate their individual constructs with their peers and the teacher within the contexts of what we call *paradigm situations*.

A *paradigm situation* is a physical situation the modeling of which requires the comprehensive use of schematic concepts in the context of one or more schematic model (Halloun 1996b; Halloun & Hestenes, 1987). Paradigm situations are sometimes presented in some textbook problems which we call *paradigm problems.* A paradigm problem has the characteristics outlined in Figure 9. In this figure, a star following a number indicates a characteristic of a higher order paradigm problem. It is not necessary that every paradigm problem have such a characteristic. However, students should have experience with enough problems to cover all the characteristics listed in Figure 9, in order to develop meaningful and comprehensive understanding of the targeted concepts and models. An exemplary paradigm problem of Newtonian mechanics with a detailed solution from a schematic modeling perspective is presented in an earlier paper (Halloun, 1995). Wells et al. (1995) provide ample details about classroom activities that can be conducted for modeling a similar situation.

Insert Figure 9 about here

Paradigm situations should be *motivating*. They should, as much as possible, bear directly on everyday life so that students realize the personal relevance of the subject matter. They should also be challenging and self-rewarding by imposing new demands on students and stimulating their creativity. Furthermore, tests and other assessment activities should be designed by teachers and viewed by students as *dialectical* diagnostics. They should help individual students ascertain and refine their own knowledge (Duschl & Gitomer, 1991). They should not serve as intimidating, ranking tools that show how much a student knows by comparison to the teacher and some peers.

Modeling paradigm situations is optimized with the integration of modern technology, and especially computers. Computers must be integrated in instructional design not as luxury tools, but as efficient learning tools. When used properly, these tools can enhance the quality of interaction among students and between them and the real world (Redish, Saul & Steinberg, 1997; Thornton & Sokolof, 1990). In data collection and analysis, for example, computers can reduce significantly the time and logistics required for such tasks. Moreover, when appropriate modeling software becomes available, computers are expected to help students extract appropriate schemas and schemes from computer-based activities (Hestenes, 1995).

Modeling paradigm situations should be done *interactively*. Individual students must interact in class with their peers as well as with their teachers. Students can then be transformed from passive recipients of canned knowledge to critical seekers and active producers of generic knowledge. The interaction is optimized when students cooperate in heterogeneous groups, inside as well as outside the classroom along the guidelines recommended by Hake (1992, 1996) and Heller et al. (1992).

Group interaction should emulate scientists' group work. It is important for students to learn how to design, carry out and evaluate a research project. But it is at least equally important that students develop the rules and ethics of team work, and get sensitized to value such work and stand accountable for it. They should appreciate the value of open minded debate, and learn how to defend their own position, how to challenge others' positions, and how to slip in others' shoes so that they can see the pro's and con's of any argument both from their own perspective and others'.

The teacher's role is central in the proposed pedagogical framework. S/he has to *mediate* learning at different levels: from putting groups together to coaching their work; from scaffolding novel scientific constructs to prompting individual students and groups with questions that guide them through the cognitive processes outlined in Figure 8 and help them organize their knowledge coherently around basic schematic models; from designing authentic assessment instruments to consulting with individual students on their progress, and so forth. In short, there is no learning without teaching; but for teaching to result in meaningful learning, it has to avoid episodic lecturing and promote schematic modeling in a student-centered approach.

Developing the Schematic Force Concept

Preliminary Results and Implications

Two groups of Lebanese students, one high school group and one college group, participated in special tutorials that followed the schematic modeling approach. Each group tutorials consisted of five two-hour sessions. During each session, students were engaged in group discussions guided by this author along the lines of Figure 8, and bearing on the solution to a number of paradigm problems that require the use of the schematic force concept in the context of some basic Newtonian models (Figure 2). The conceptual evolution of participating students was assessed based on their solutions to parallel paper-and-pencil paradigm problems administered to them before and after the tutorials. By comparison to their performance on the respective pretests, both groups showed significant improvement on their posttests. Procedures and results of these tutorials are reported elsewhere in detail (Halloun, 1996b). In the following, we briefly review and discuss some of these results that pertain to the schematic dimensions of the concept of force.

Domain

A major deficiency noticed in the pretests was students' inability to isolate objects from their agents, and identify the actual forces involved in a given physical situation. During the tutorials, a special care was given to help students realize the importance of identifying the agents acting on individual objects (Figure 3) before trying to list the possible forces and do any manipulations with them. Consequently, on the posttests, at least 80% of participating students followed this strategy in any given problem and were able to identify and depict the correct forces acting on individual objects. All students who failed to identify agents first, and those who committed mistakes while doing so drew wrong force diagrams. Furthermore, on the pretests, students often showed superfluous forces that actually did not exist; virtually no student in either group did so on the posttests.

Expression

On the pretests, most students did not attempt to draw force diagrams depicting the interactions involved, and most of those who did make such an attempt were not successful in doing so. During the tutorials, students were guided to draw such diagrams systematically. In situations requiring particle models (Figure 2), a special care was given to convince students to draw force depicting arrows with all their tails coinciding on the point representing the particle-like object. This was done to emphasize two facts. The first is that, unlike the case with rigid bodies in rotation, one can ignore the shape of an object in translation and the point of application of a given force. The second is that vectorial operations are easier to conceive when conducted with a diagram whereby all arrows emerge from the same point. Furthermore, students were encouraged to draw an arrow depicting the acceleration of a particle in a given reference frame along with the corresponding force diagram. All students who followed this strategy on the posttest (69% to 88% on the various problems) were able to depict forces in correct force diagrams, especially forces that could be identified by only 4% of college students in a control group.

Quantification

The majority of participating students was unable on the pretests to quantify forces correctly as vectorial quantities. Judging from the success on the posttests in showing appropriate force diagrams as reported above, and in attempting adequate mathematical operations as described below, most participating students were able to conceive the vectorial nature of the force concept following the tutorials.

Organization

Most students' solutions on the pretest problems consisted exclusively of writing and solving sets of mathematical equations that most often depicted wrong models or wrong laws for a given physical situation. During the tutorials, students were guided to isolate individual objects in a given physical situation, and identify the appropriate model for each object in a convenient reference frame, before attempting any mathematical manipulations. On the posttests, all college students (72% to 88%) and 88% of high school students (53% to 69%) who followed this strategy successfully were able to construct a complete mathematical model (force diagrams, equations and justification) that correctly depicts the situation at hand.

Employment

On the pretests, no more than 5% of high school students and 20% of college students were able to solve any administered problem correctly. On the posttests, the rate of success rose to 34% in high school, and 72% in college (as opposed to only 1% in a control college group that was administered the same posttest). Students who were able to set up the correct mathematical model but failed to complete the solution of a problem did so mainly because they failed to resolve a force vector into components or evaluate these components, and/or to compose many force vectors or evaluate their resultant. Failure to produce a complete solution to a problem was then often largely due to deficits in mathematical skills than any other schematic aspect of the force concept.

The dialectical approach

The relative success of the tutorials depended to a large extent on guiding students to develop the schematic dimensions of the force concept in the context of basic Newtonian models. Another major factor in this success was the active engagement of students in team work and group discussions whereby interested students were afforded the chance of negotiating their own constructs with their peers in the classroom. As to which of the two factors and which of the epistemological or cognitive elements affected the results most, the experiment was not controlled enough to answer this question. However, and judging from our own and other people experiences, we can safely conclude that both factors and each of their elements had their positive contributions.

The tutorials described here represent only a partial and preliminary application of the schematic modeling approach. Their results are by no means the final substantial evidence for the validity of this approach. However, they provide some insights into the potentials of schematic modeling, and some motivation to pursue research along its lines.

Further research is required to assess individual dimensions of schematic concepts and models, individual stages of the proposed scheme for modeling paradigm situations, the characteristics of such situations, as well as of the cognitive underpinnings of the proposed approach. Such a research project is extremely broad and is well beyond the scope not only of this paper, but of any single researcher or research group for that matter.

Various forms of model-based instruction have been pursued by other researchers referred to in the introduction. Schematic modeling is original, especially in its epistemology. In this respect, and so far as this author knows, this is the first attempt to propose specific dimensions that could serve as generic templates for defining schematic concepts and models.

Numerous are the calls to reform the content of science courses and of our methods of instruction. However, no alternative approach has been proven yet to improve significantly the state of science education anywhere in the world. Schematic modeling is an ambitious project in this direction.

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Uniformly Accelerated Particle Model

1. Domain

All physical systems that are in *translation* with *constant acceleration* in inertial reference systems. Each system is interacting with one or many physical agents that exert on it a net *constant force*.

The model could be kinematical (descriptive, not involving forces), and/or dynamical (explanatory, involving forces), within the framework of Newtonian Theory.

2. Composition

2.a Content: *One* particle representing a system in consideration, and depicted by a geometric point in a coordinate system representing the reference frame.

2.b Environment: Two types of agents are distinguished in Newtonian mechanics, those of interaction at-a-distance (or long-range interaction), and those of contact interaction. Figure 3 below shows a number of typical agents encountered in introductory mechanics courses.

2.c Object descriptors refer to *intrinsic* and *state properties* of the particle-like system. Only one intrinsic property is accounted for in any particle model: the mass of an object. State properties are the kinematical properties of the object. They include position, displacement, velocity, acceleration, kinetic energy, etc.. In a coordinate system, state properties are depicted in a *motion map* consisting of a trajectory on which can be shown position, velocity and acceleration vectors at instants of interest.

2.d Interaction descriptors include, among others, the concept of force. Forces exerted on the particle-like system by its agents are often depicted by arrows in a *force diagram*. The net force is evaluated using the superposition principle (Newton's 4th law or law of composition).

3. Structure

3.a Geometric structure, if any in this model, is often restricted to the relative position of the particle-like object and long-range agents.

3.b Interactive structure, expressed by appropriate interaction laws, like the Newtonian law of universal gravitation.

3.c Behavior description: The translation of the particle is described by *motion laws* (state laws) involving only kinematical concepts, such as:

$\mathbf{a}$ = constant $\qquad \Delta\mathbf{v} = \mathbf{a}t \qquad \Delta\mathbf{r} = \mathbf{v_o}t + \frac{1}{2}\mathbf{a}t^2 \qquad \Delta\mathbf{v}^2 = 2\mathbf{a}\Delta\mathbf{r}$

These laws can also be depicted by appropriate *graphs*, *relational diagrams* and/or *motion maps*.

3.d Behavior explanation: Variations of the particle's state properties are explained by *causal laws* such as Newton's 2nd law (dynamical law) or the Work-Energy theorem (conservation law).

4. Organization

Newtonian Theory relates the uniformly accelerated particle model to:

- other models within the family of particle models (Fig. 1).
- rigid body models and other families of models (e.g. fluid models) within the theory.

Consequent rules are established within the theory that tell us how to combine this model with other basic models to study physical systems undergoing more complex motions. For example, this model can be combined with the uniformly circling particle model to describe, explain, and/or predict the behavior of an object that undergoes a circular, uniformly accelerated translation. If the object were also rotated about a specific axis (like in the case of Earth), the *emergent* model thus constructed could be further combined with the appropriate rigid body model.

Figure 1: Schematic dimensions of the Newtonian uniformly accelerated particle model.

Basic Particle Models

Particle models refer to physical objects the internal structure of which can be ignored when they are in *translation* without rotation or precession, in a specific reference system. The content of each *basic* particle model consists of a single, dimensionless object: a particle. Basic particle models include:

Free Particle

This model refers to physical objects subject to zero net force ($\sum \mathbf{F}_i = 0$) in linear translation with constant velocity or at rest.

Uniformly Accelerated Particle

This model refers to physical objects subject to a net constant force ($\sum \mathbf{F}_i$ = constant), hence moving with constant acceleration in a linear or parabolic path.

Harmonically Oscillating Particle

This model (often called simple harmonic oscillator) refers to physical objects subject to a net force that is proportional to their displacement from a center of force ($\sum \mathbf{F}_i \propto \Delta \mathbf{r}$), hence undergoing simple harmonic motion.

Uniformly Circling Particle

This model refers to physical objects subject to a net centripetal force ($\sum_{radial} \mathbf{F}_i \propto 1/r^2$) of constant magnitude, hence undergoing a uniform circular motion.

Basic Rigid Body Models*

Rigid body models refer to physical objects the internal structure of which cannot be ignored when they are in *translation* and/or *rotation* and/or *precession*. The content of each *basic* rigid body model consists of one solid of a regular geometric shape rotating (precession ignored) about a specific axis (that may be fixed or in translation, and that can be modeled like a particle). Basic rigid body models include:

Freely Rotating Rigid Body

This model refers to physical objects subject to zero net torque about a specific axis ($\sum \tau_i = 0$), hence rotating with constant angular velocity about this axis or at rest.

Uniformly Accelerated Rotating Rigid Body

This model refers to physical objects subject to a net constant torque about a specific axis ($\sum \tau_i$ = constant), hence rotating with constant angular acceleration about this axis.

Figure 2: Two families of basic models in Newtonian theory (Halloun, 1996b).

* Euler's laws (which are sometimes wrongly referred to as "Newton's laws of rotation") govern the rotation of rigid body models. Euler's laws follow the Newtonian philosophy. In this respect, the family of rigid body models is here considered part of the "Newtonian" theory. One, though, could still refer to "Newtonian Theory" in the narrower sense, and restrict it to particle models.

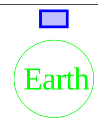
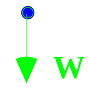
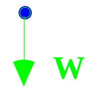
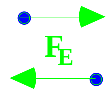
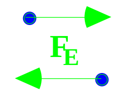
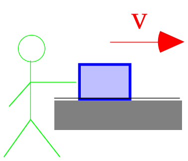
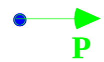
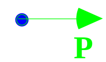
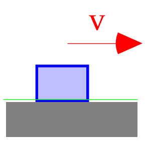
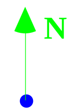
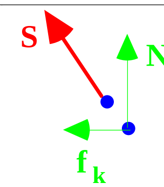
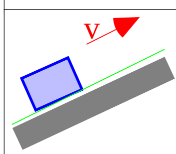
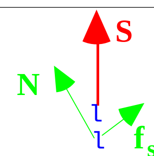
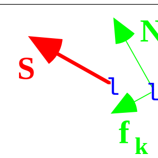
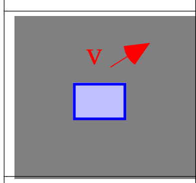
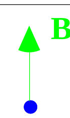
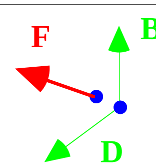
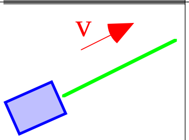
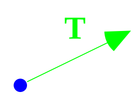
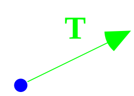
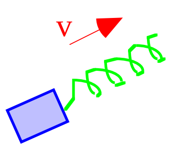
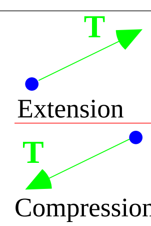
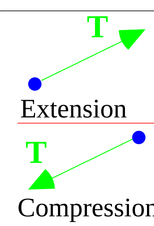
	Agent		Force	Force Diagram	
	Name	Examples	Name	Rest	Motion
Long-range Interactions					
	Any physical object	Earth, Moon, Sun, other planets	Gravitational force or Weight **W**		
	Electric charge carriers	Electrons, protons, ions, macro-objects	Electrostatic force $\mathbf{F_E}$		
Contact Interactions					
	Direct mover	Human hand, one car directly pushing another car	Traction: Push, pull **P**		
	Horizontal Solid Support	Table, ground, road, shelf, board, human hand	Support force **S**. Components: Normal **N** & friction **f**		
	Inclined Solid Support	Table, ground, road, shelf, board, human hand	Support force **S**. Components: Normal **N** & friction **f**		
	Fluid	Air, water, other gas / liquids	Fluid force **F**. Components: Buoyancy **B** & drag **D**		
	Rigid Suspender	Rigid rope, string, rod, bar, or chain, human arm	Tension **T**		
	Elastic Suspender	Spring, Elastic rope, string, rod, bar, or chain	Restoring force **T**		

Figure 3: Sample force catalog for Newtonian particle models (Halloun, 1996b).

Force Domain

- The *domain* of the concept of force consists of all couples of interacting physical objects. In Newtonian models, the couple's element that is being studied is referred to as the *object*, the other as the *agent*.
- An object can *not* interact with itself. Every force must have an external agent. Unless a distinct agent exists that interacts in a specific way with a given object, the concept of force cannot be used.
- The concept of force is *explanatory*. It is a concept of dynamics and not kinematics; it explains the *change* in the momentum (or velocity) of an object.
- The existence of an interaction, and hence the need for the concept of force, can be recognized from the *kinematical state* of an object: a free particle needs not to interact with any agent to maintain its constant momentum (or velocity); however any *change* in its momentum requires an interaction with one or many agents.
- A single force represents *one side* of the interaction, the action of an agent on an object, or that of the latter on the former.
- Forces come in pairs: the two opposite forces exchanged by an object and an agent are *simultaneous,* and both are involved in any interaction.
- *No intermediary* between an object and an agent is needed for them to interact (this is true at the macroscopic level but not necessarily at the microscopic level).

Figure 4: Some correspondence rules for the Newtonian concept of force.

Force Quantification

Quantification laws

- Force is a *vectorial* concept (as opposed to *scalar*, like the concepts of mass or temperature), and hence its measurement requires the specification of a direction (Figure 3), a magnitude and a unit which is the Newton (N) in SI.
- Force is an *extensive* concept (as opposed to *intensive*, like temperature), i.e., a single force of magnitude zero indicates no net interaction.
- Force is an *additive* concept (as opposed to *non-additive*, like temperature); two or more forces can be added vectorially following the superposition principle.
- Force is a *proportional* concept (as opposed to *ordinal,* like temperature); two forces can be compared by a ratio.
- A force is indirectly measured physically; there are no direct means (or physical probes) for comparing a given force to a standard force in the same way, say, the length of an object is physically measured by comparing it to the graduation of a ruler. A force is always measured through its effect on a given object, like stretching or compressing a spring.

  Thus, there are some *assumptions* underlying the *measurement* of a force. For example, changing the strength of an interaction between an object and an agent is assumed to induce a proportional change in a given state property of the object. Two forces are then axiomatically said to have equal magnitudes if they produce the same effect on the same object (which further assumes that after each measurement, the object can be brought back exactly to the same initial conditions).

Quantification rules

These rules set among others:

- How to set the dimension of a force, as given symbolically by:

  $$[\text{Force}] = \frac{[\text{Mass}]\times[\text{Length}]}{[\text{Time}]^2}$$

- How to convert from the SI to the cgs or the old British unit systems, knowing that:
  1 dyne = 10^{-5} N (cgs) and 1 lb = 4.448 N (Brit.).
- How to determine the characteristics of a force exerted by a given agent (Figure 3).
- How to measure a force physically using appropriate force probes (e.g., spring scales), and establish the correspondence between “reading” an effect and the magnitude of the force that causes it.
- How to estimate errors in an experimental setting.

Figure 5: Quantification laws and rules associated with the Newtonian concept of force.

Force Expression

Means of expression

These include:

- *Identification* of the concept, i.e., its name (force) and the name(s) of its unit(s) (Newton or dyne), all of which are particular to this concept and cannot be shared by other concepts.
- *Symbolic labels*, i.e., specific characters that can denote the concept or its units instead of their names, and the appropriate style:

  Force is a vectorial concept that can be denoted by a bold letter (e.g., **F**) or a letter with a small arrow on top (e.g., $\vec{F}$).
- *Pictorial depictions*, i.e., geometric figures that can depict the concept:

  A force is depicted by a vector, a labeled arrow, in an appropriate coordinate system (Figure 3). Specific assumptions underlie the point of application of this vector, depending on whether or not the object is particle-like.
- *Mathematical representations*, including equations, graphs, and geometric diagrams representing the concept and its relation to other concepts.

Semantics

These specify, among others:

- What each form of expression denotes, especially that each form can denote specific features of a concept but never all its features:

  A normal letter labels the magnitude of a force, whereas a bold letter labels its direction as well.
- How to *interpret* each form of expression and establish the appropriate correspondence to the real world:

  The magnitude and direction of interaction between a physical object and agent can be determined appropriately from the corresponding force vector.

  The equality in "**F** = m**a**" relates a force **F** exerted by an *agent* to its effect **a** on an *object* of mass m, and expresses a different relationship from the one expressed, say, in "**a** = d**v** / dt" for "defining" the acceleration of an object in terms of its own velocity.
- How different forms of expression relate to, and complement, each other in specific respects:

  A force vector can only depict a force at a given instant. Changes in its direction and magnitude may be better represented by appropriate diagrams such as field lines, graphs and/or equations.

Figure 6: Means of expression and semantics of the Newtonian concept of force.

Force Employment

What to do when studying particle interaction?

- Set convenient *system boundaries* in a conveniently chosen inertial reference system, so that every system can be represented by a *particle model*.
- Depict the reference system by a convenient coordinate system, and the particle by a point in this system.
- Identify agents, remembering that, except for the Earth and electrically charged particles, no physical entity can be an agent unless it is in contact with a given object.
- Identify the force exerted by each agent on a given object.
- Depict every force by an appropriate vector in a force diagram, with the tails of all force vectors coinciding at the point depicting the particle.
- Resolve a force vector into appropriate components.
- Compose many force vectors following the superposition principle.
- Match various mathematical representations of a force, and conduct appropriate operations with those representations.
- Match the resultant force on an object with the acceleration of the object.
- Choose between Newton's laws and the work-energy principle to relate the resultant force to its effect on a given object.

Figure 7: Guidelines for employing the Newtonian concept of force in basic particle models.

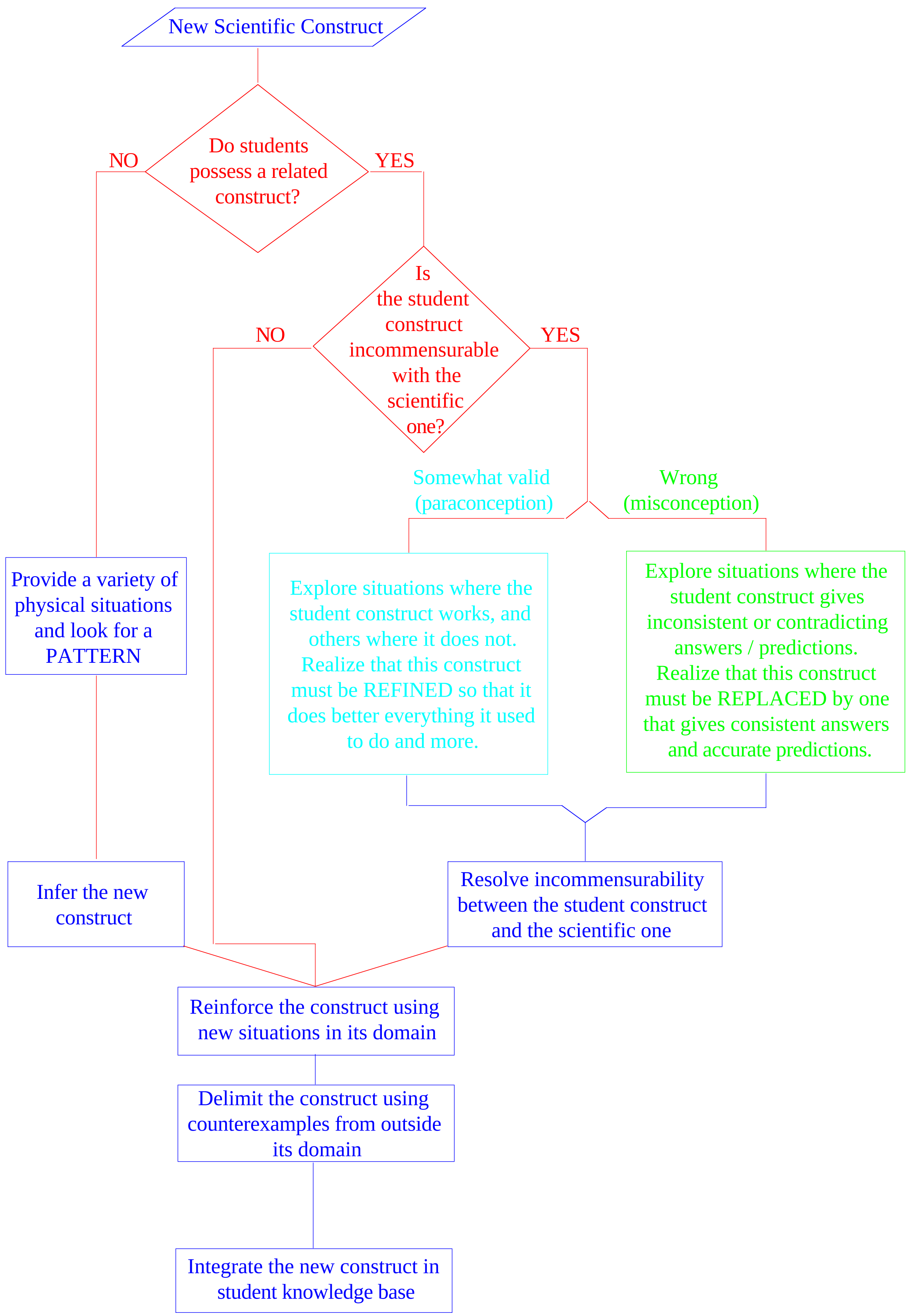


Figure 8: Toward commensurability between student and scientist constructs.

Paradigm Problems

1. A paradigm problem is not a straightforward numerical application of *formulas*.
2. The problem describes a real world situation with physical objects.
   In order to reinforce the universality of models, and hence of physics theories, different problems that can be solved using the same model should be presented that involve different contexts, or objects of different scale (from common ones in everyday life to microscopic or astronomical ones).
3. The problem involves some *composite* system (made out of at least two interacting objects), or more than one *simple* system.
4. The problem does not suggest explicitly the appropriate model for each object/event.
5. * Different objects may undergo events of different types (e.g. translation in one or two dimensions, and/or rotation), and hence require models from different families.
6. * Construction of at least one new emergent model is required, out of familiar ones.
7. * A model can be constructed out of the *givens* without the *question*(s).
8. * The problem contains superfluous information.
9. The problem contains constraints, limits or boundary conditions.
10. * Some required information is not provided (aside from familiar constants, like g), such as the direction of motion, in an Atwood machine, or the direction of current in an electric circuit.
11. Model composition includes descriptors of different types: e.g. descriptive (kinematics) and explanatory (dynamics).
12. Model structure requires a non-straightforward choice of appropriate laws or the use of many laws (e.g., Newton's laws and W-E Theorem).
    As in (2), and in order to foster the unification of physical theories, questions are included that require generic laws like the superposition principle, and conservation laws.
13. Model construction involves multiple mathematical representations that need to be extrapolated and coordinated: e.g., diagrams, graphs and equations. However, mathematical operations should be kept at a minimum.
    Numerical calculations should not be tedious, and should have a conceptual purpose if required (e.g., to establish correspondence to the real world, and facilitate extrapolation of results).
14. * Questions do not specify explicitly concepts that need be evaluated (e.g. it is better to ask *where* two objects *meet* than to ask for the common *position* at the meeting time).
15. * Questions ask for a comparison of objects with respect to a specific property (preferably not stated explicitly) rather than for an evaluation of this property for each object separately (e.g., it is better to ask "*how would two objects see each other moving?*" than to ask for the velocity/acceleration of each).
16. Follow-up questions are included that ask for real world interpretations, and that help to resolve common misconceptions.
17. Follow-up questions are included that ask for results extrapolation (e.g., predict what happens if something changes in the situation, or if we look at the same situation in a different reference system).

Figure 9: Major characteristics of a paradigm problem.

* A characteristic of a higher order paradigm problem. See text for details.